UNIT FOUR
Wave Motion and Optics

Wave motion describes phenomena ranging from the familiar...

Ocean waves....
Sound waves...
Light waves...

... to the less familiar realm of atomic physics...
...even
Quantum
Physics
(matter
waves)...
... and Relativity (gravity waves).
Chapter 15
Making Waves
Water seems to move toward the shore, but no water accumulates on the beach.

What is actually moving?

Do the waves carry energy?
Wave Pulses and Periodic Waves

A Slinky is ideal for studying simple waves.
If a Slinky is laid out on a smooth table with one end held motionless, you can easily produce a single traveling pulse:

- With the Slinky slightly stretched, move the free end back and forth once along the axis of the Slinky.
- You will see a disturbance (the wave pulse) move from the free end of the Slinky to the fixed end.

What is actually moving?

- The pulse moves through the Slinky, and portions of the Slinky move as the pulse passes through it.
- After the pulse dies out, the Slinky is exactly where it was before the pulse began.
Moving one end of the Slinky back and forth created a local compression where the rings of the spring are closer together than in the rest of the Slinky.

- This region of compression moves along the Slinky and constitutes the pulse.
- The wave or pulse moves through the medium (here, the Slinky), but the medium goes nowhere.
- What moves is a disturbance within the medium which may be a local compression, a sideways displacement (like a wave on a rope), etc.

The speed of the pulse may depend on factors such as tension in the Slinky and the mass of the Slinky.
Energy is transferred through the Slinky as the pulse travels.

- The work done in moving one end of the Slinky increases both the potential energy of the spring and the kinetic energy of individual loops.
- This region of higher energy then moves along the Slinky and reaches the opposite end.
- There, the energy could be used to ring a bell or perform other types of work.

Energy carried by water waves does substantial work over time in eroding and shaping a shoreline.
The pulse we have been discussing is a **longitudinal wave**: the displacement or disturbance in the medium is **parallel** to the direction of travel of the wave or pulse.

Sound waves are longitudinal.
By moving your hand up and down, you could also produce a **transverse wave**, in which the displacement or disturbance is **perpendicular** to the direction the wave is traveling.

Waves on a rope and electromagnetic waves are transverse.

Polarization effects are associated with transverse waves but not longitudinal waves.

Water waves have both longitudinal and transverse properties.
If instead of moving your hand back and forth just once, you continue to produce pulses, you will send a series of longitudinal pulses down the Slinky.

- If equal time intervals separate the pulses, you produce a periodic wave.
- The time between pulses is the period $T$ of the wave.
- The number of pulses or cycles per unit of time is the frequency $f = 1/T$.
- The distance between the same points on successive pulses is the wavelength $\lambda$.
- A pulse travels a distance of one wavelength in a time of one period.
- The speed is then the wavelength divided by the period:

$$v = \frac{\lambda}{T} = f \lambda$$
A longitudinal wave traveling on a Slinky has a period of 0.25 s and a wavelength of 30 cm. What is the frequency of the wave?

- a) 0.25 Hz
- b) 0.30 Hz
- c) 0.83 Hz
- d) 1.2 Hz
- e) 4 Hz

\[
T = 0.25 \text{ s} \\
f = \frac{1}{T} = \frac{1}{0.25 \text{ s}} = 4 \text{ Hz}
\]
A longitudinal wave traveling on a Slinky has a period of 0.25 s and a wavelength of 30 cm. What is the speed of the wave?

a) 0.25 cm/s  
b) 0.30 cm/s  
c) 1 cm/s  
d) 7.5 cm/s  
e) 120 cm/s

\[ v = f \lambda \]

\[ = (4 \text{ Hz})(30 \text{ cm}) \]

\[ = 120 \text{ cm/s} \]
Waves on a Rope

- A snapshot of a single transverse pulse moving along a rope is like a graph of the vertical displacement of the rope plotted against the horizontal position.

- At some later time the pulse will be farther down the rope at a different horizontal position.
- The shape remains basically the same.
If you repeat a series of identical pulses at regular time intervals, you might produce a periodic wave such as shown.

- The wavelength $\lambda$ is the distance covered by one complete cycle of the wave.
- This wave pattern moves to the right along the rope, retaining its shape.
- The shape depends on the exact motion of the hand or other oscillator generating the wave.

When the leading edge of the wave reaches the fixed end of the rope, it will be reflected and start to move back to the left.

- The reflected wave will interfere with the wave still traveling to the right.
If you move your hand up and down smoothly in simple harmonic motion, the displacement of this end of the rope will vary sinusoidally with time.

- The resulting periodic wave will also have a sinusoidal form.
- Such a wave is called a **harmonic wave**.
- The individual segments of rope tend to move with simple harmonic motion, because the restoring force pulling the rope back toward the center line is proportional to its distance from the center line.

Any periodic wave can be represented as a sum of harmonic waves with different wavelengths and frequencies.

- The process of breaking a complex wave down into its simple harmonic components is called **Fourier**, or **harmonic analysis**.
A wave on a rope is shown below. What is the wavelength of this wave?

- a) 1/6 m
- b) 1 m
- c) 2 m
- d) 3 m
- e) 6 m

In 6 m, the wave goes through 2 complete cycles. The wavelength (length of one complete cycle) is $(6 \text{ m})/2 = 3 \text{ m}$. 
If the frequency of the wave is 2 Hz, what is the wave speed?

\[ v = f \lambda = (2 \text{ s}^{-1})(3 \text{ m}) = 6 \text{ m/s} \]

- a) 1/6 m/s
- b) 2/3 m/s
- c) 2 m/s
- d) 3 m/s
- e) 6 m/s
As the raised portion of a pulse approaches a given point on the rope, the tension in the rope acquires an upward component.

- The resulting upward force causes this next segment to accelerate upward, and so on down the rope.

- The speed of the pulse depends on how fast succeeding segments can be started moving (accelerated).

  - By Newton’s second law, this is proportional to the force and inversely proportional to the mass of the segment:
    \[ a = \frac{F}{m} \]
    A larger tension produces a larger acceleration.
    The speed of the pulse will increase with the tension and decrease with the mass per unit length of the rope:
    \[ v = \sqrt{\frac{F}{\mu}} \]
    where \( \mu = \frac{m}{L} \)
A rope has an overall length of 10 m and a total mass of 2 kg. The rope is stretched with a tension of 50 N. One end of the rope is fixed, and the other is moved up and down with a frequency of 4 Hz. What is the speed of waves on this rope?

a) 5.0 m/s  b) 7.07 m/s  c) 15.8 m/s  d) 50 m/s  e) 250 m/s

\[ \mu = \frac{m}{L} = \frac{2 \text{ kg}}{10 \text{ m}} = 0.2 \text{ kg/m} \]

\[ \nu = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{50 \text{ N}}{0.2 \text{ kg/m}}} = \sqrt{250 \text{ m}^2/\text{s}^2} = 15.8 \text{ m/s} \]
A rope has an overall length of 10 m and a total mass of 2 kg. The rope is stretched with a tension of 50 N. One end of the rope is fixed, and the other is moved up and down with a frequency of 4 Hz. What is the wavelength?

a) 0.20 m  b) 3.95 m  c) 10 m  d) 15.8 m  e) 25 m/s

\[ \nu = f \lambda \Rightarrow \]
\[ \lambda = \frac{\nu}{f} = \frac{15.8 \text{ m/s}}{4 \text{ Hz}} = 3.95 \text{ m} \]
Interference and Standing Waves

- When a wave on a rope reaches the fixed end of the rope, it is reflected and travels in the opposite direction back toward your hand.
  - If the wave is periodic, the reflected wave interferes with the incoming wave.
  - The resulting pattern becomes more complex and confusing.
  - This process, in which two or more waves combine, is called interference.
Imagine a rope consisting of two identical segments spliced together to form a single rope of the same mass per unit length as the two original segments.

- Identical waves traveling on the two identical segments will combine to form a larger wave on the single joined rope.
- At all points, the height of the individual waves will add together to form a wave with the same frequency and wavelength but twice the height of the initial two waves.

**Principle of Superposition:**
When two or more waves combine, the resulting disturbance or displacement is equal to the sum of the individual disturbances.
When the two waves are moving the same way at the same time, they are *in phase*.

- The resulting combined wave will be larger (have a greater height).

If one wave is moving upward when the other wave is moving downward, the two waves are *completely out of phase*.

- If the two waves have the same height, the resulting combined displacement will be zero.
- No wave is propagated beyond the junction.

The result of adding two waves together depends on their *phases* as well as on their height or *amplitude*.

- When waves are in phase, we have constructive interference.
- When waves are out of phase, we have destructive interference.
When two or more waves are traveling in the same direction, the difference in phase determines whether the interference will be constructive, destructive, or somewhere in between.

When two waves are traveling in opposite directions, such as when a wave is reflected back on itself, the principle of superposition can be applied at different points on the string.

- At point A, the two waves cancel each other at all times.
- At this point, the string will not oscillate at all; this is called a node.

At point B, both waves will be in phase at all times.
The two waves always add, producing a displacement twice that of each wave by itself.
This is called an antinode.
This pattern of oscillation is called a *standing wave*.  
- The waves traveling in opposite directions interfere in a way that produces a standing or fixed pattern.  
- The distance between adjacent nodes or adjacent antinodes is half the wavelength of the original waves.  
- At the antinodes, the string is oscillating with a large amplitude.  
- At the nodes, it is not moving at all. 
- At points between the nodes and antinodes, the amplitude has intermediate values.
Guitars, pianos, and other stringed instruments produce music using standing waves on strings.

- The frequency of the sound wave equals the string’s frequency of oscillation and is related to the musical pitch.
- A higher frequency represents a higher-pitched sound.
For a string fixed at both ends, the simplest standing wave, the **fundamental** or **first harmonic**, has nodes at both ends and an antinode in the middle.

- The wavelength is determined by the length of the string.
- Since the distance between nodes is half the wavelength, the wavelength must be twice the length of the string.
- The wave speed is determined by the tension in the string and the mass per unit length of the string.
- The frequency can then be found using the relationship $v = f\lambda$:

$$f = \frac{v}{\lambda} = \frac{v}{2L}$$
- A string with a longer length $L$ will result in a lower frequency.
  - The effective length can also be shortened by placing your finger firmly on the string, producing a higher-pitched tone.
- Other patterns of oscillation may also be produced.
  - The second harmonic has a node at the midpoint of the string, and a wavelength equal to $L$.
  - This wavelength is half the fundamental, so its frequency is twice the fundamental.
  - Musically, this pitch would be an octave above the fundamental.
A string with a longer length \( L \) will result in a lower frequency.
- The effective length can also be shortened by placing your finger firmly on the string, producing a higher-pitched tone.

Other patterns of oscillation may also be produced.
- The third harmonic has four nodes (counting the ones at the ends) and three antinodes, and a wavelength equal to two-thirds \( L \).
- The resulting frequency is three times the fundamental and \( 3/2 \) that of the second harmonic.
- Musically, this is called a fifth above the second harmonic.
A guitar string has a mass of 4 g, a length of 74 cm, and a tension of 400 N. These values produce a wave speed of 274 m/s. What is its fundamental frequency?

\[ L = 74 \text{ cm} = 0.74 \text{ m} \]
\[ v = 274 \text{ m/s} \]
\[ \lambda = 2L \]
\[ f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \]
\[ = \frac{274 \text{ m/s}}{1.48 \text{ m}} \]
\[ = 185 \text{ Hz} \]

a) 1.85 Hz
b) 3.70 Hz
c) 185 Hz
d) 274 Hz
e) 370 Hz
A guitar string has a mass of 4 g, a length of 74 cm, and a tension of 400 N. These values produce a wave speed of 274 m/s. What is the frequency of the second harmonic?

\[ L = 74 \text{ cm} = 0.74 \text{ m} \]
\[ v = 274 \text{ m/s} \]
\[ \lambda = L \]
\[ f_2 = \frac{v}{\lambda_2} = \frac{v}{L} \]
\[ = \frac{274 \text{ m/s}}{0.74 \text{ m}} \]
\[ = 370 \text{ Hz} \]

a) 92.5 Hz  
b) 123 Hz  
c) 185 Hz  
d) 370 Hz  
e) 740 Hz
Sound Waves

- Sound waves may be generated in many ways in addition to an oscillating string.
- Since sound waves reach our ears, they must be able to travel through air.

The bass string has been plucked, producing a blur near the middle where the amplitude is greatest.
A sound wave consists of pressure variations in air.

- The diaphragm of a speaker oscillates back and forth, producing regions of higher pressure and lower pressure.
- These regions propagate through the air as variations in air pressure and density, forming a *longitudinal* sound wave.
We can show these pressure variations in a graph of pressure plotted against position.

The factors that determine the speed of sound are related to how rapidly one air molecule transmits changes in velocity to nearby molecules to propagate the wave.

- In room temperature air, sound waves travel with a speed of 340 m/s or 750 MPH.
- For gases other than air, the speed also depends on the masses of the molecules or atoms.
- Sound waves can also travel through liquids and solids, often with higher speeds.
Interference phenomena such as standing waves can be observed in sound waves.

- Many musical instruments produce standing waves in a tube or pipe.
- If the tube is closed at one end, such as a bottle, there is a **displacement node** at the closed end.
- At the open end, there is a displacement antinode.

The frequency of the standing wave can be found from the speed of sound in air and the wavelength:

\[
f = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{\lambda}
\]

where the wavelength is determined by the length of the tube.
The standing-wave patterns for the first three harmonics for a tube open at one end and closed at the other are represented as follows:

- The first harmonic or fundamental has a wavelength four times longer than the length of the tube.
- The wavelength of the second harmonic is equal to four-thirds of the length of the tube.
- The wavelength of the third harmonic is equal to four-fifths of the length of the tube.
- *etc.*

\[
f = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{\lambda}
\]
The Doppler Effect

- A moving source of sound, such as a car horn, seems to change pitch depending on its motion relative to the listener.

  - As a car passes a stationary observer, the horn’s pitch changes from a higher pitch to a lower pitch.
The Doppler Effect

- Comparing the wavefronts for a stationary car horn and for a moving car horn illustrates why the pitch changes.
  - When the car is approaching the observer, the wavefronts reaching the observer are closer together.
  - When the car is moving away from the observer, the wavefronts reaching the observer are farther apart.
The Physics of Music

- Why do certain combinations of notes (chords) sound better than others?
  
  - Musical notes actually have a mix of higher harmonics along with the fundamental frequency.
  
  - The relationships between these different frequencies explain why some notes sound harmonious together, and others do not.
  
  - An analysis of the mixture of frequencies is called *harmonic* or *frequency analysis*.
When a guitar string is plucked in the usual position, the second and third harmonics often dominate the harmonic spectrum.

- $f_1$ is the frequency of the first harmonic.
- Since the string is usually plucked right where the second harmonic ($f_2 = 2f_1$) has an antinode, the second harmonic is strongly stimulated.
- The body of the guitar also determines which harmonics will be reinforced.
When a guitar string is plucked near the bridge, many higher harmonics are present in the harmonic spectrum.

- This results in a twangy sound.
- The pitch still sounds the same, but the tone quality depends on the mixture of harmonics present.

Different instruments generate different harmonic mixes, so they have different tones.

- A trumpet produces a lot of higher harmonics so it sounds “bright” or “brassy”.
- A flute can produce a tone dominated by the fundamental frequency, with almost none of the higher harmonics, so the tone sounds “pure”.

![Harmonic spectrum graph](image-url)
When discussing musical notes, we describe the difference in frequencies in terms of **musical intervals**.

Musical scales and intervals are based upon the ratios between the higher harmonics in the notes.

- The frequency of the second harmonic is twice that of the first harmonic. This interval is an **octave**, and corresponds to the interval from one note C to the next higher C (the first and eighth notes in the scale).
- The frequency of the third harmonic is 3/2 that of the second harmonic. This interval is a **fifth**, between the first and fifth notes in the scale (C and G).
- The frequency of the fourth harmonic is 4/3 that of the third harmonic. This interval is a **fourth**, between the first and fourth notes (C and F).
- The frequency of the fifth harmonic is $5/4$ that of the fourth harmonic. This interval is a *major third*, between the first and third notes (C and E).

- **Just tuning** involves tuning an instrument so that in one key, all the intervals have simple frequency ratios.
  - In a different key, the tuning may not sound correct.

- **Equally-tempered tuning** is a compromise so that the ratios are all approximately correct, but not perfect.
  - The ratios between adjacent half steps on the scale are all identical, so the scales sound correct regardless of what key you are in.
A C-major scale begins with do on middle C having a frequency of approximately 264 Hz. Assuming that they have been tuned to the perfect ratios for the intervals in question, what should the frequency be for sol (G)?

- a) 88 Hz
- b) 132 Hz
- c) 176 Hz
- d) 396 Hz
- e) 528 Hz

\[ f = \frac{3}{2} (264 \text{ Hz}) = 396 \text{ Hz} \]
A C-major scale begins with do on middle C having a frequency of approximately 264 Hz. Assuming that they have been tuned to the perfect ratios for the intervals in question, what should the frequency be for fa (F)?

a) 176 Hz  
b) 198 Hz  
c) 352 Hz  
d) 440 Hz  
e) 528 Hz

fa is a fourth above do with a ratio of $4/3$:

$$f = \frac{4}{3} (264 \text{ Hz}) = 352 \text{ Hz}$$
A C-major scale begins with do on middle C having a frequency of approximately 264 Hz. Assuming that they have been tuned to the perfect ratios for the intervals in question, what should the frequency be for do at the top of the scale (high C)?

a) 88 Hz  
b) 198 Hz  
c) 352 Hz  
d) 440 Hz  
e) 528 Hz

From do to the next higher do is an octave, which doubles the frequency:

\[ f = 2(264 \text{ Hz}) = 528 \text{ Hz} \]
- Combinations of notes sound harmonious when the higher harmonics overlap.
- When two notes are too close in pitch, **beats** can produce a dissonant buzz.
- The two waves come in and out of phase as time progresses.