Exam 1 Review

1. Give complete definitions of the following concepts:
   a. even integer

   b. odd integer

   c. $a|b$, where $a, b$ are integers

   d. perfect square

   e. $A \subseteq B$, where $A, B$ are sets

2. Prove or disprove each of the following statements. (Assume every variable used represents an integer.)
   a. The sum of two even integers is even.

   b. The product of two odd integers is odd.

c. If $a|c$ and $b|c$ then $(a+b)|c$.

d. If $2|a$ and $3|b$ then $6|(3a+2b)$.

e. If $a|b$ or $a|c$ then $a|(bc)$.

f. No integer is divisible by 0.

g. If $x$ is an integer, then $x^2+x$ is even.

3. Construct the truth table to show that the statements "If P then Q" and "Q or not P" are logically equivalent.

4. Consider the following statements:
   (A) If $3|a$ and $3|b$ then $3|(ab)$.
   (B) If $3|(ab)$ then $3|a$ or $3|b$.
   (C) If $3|(ab)$ then $3|a$ and $3|b$.
   (D) The product of two integers, neither of which is divisible by 3, cannot be divisible by 3.

   Do the following:
   a. One of the statements is false. Determine which one and give a counterexample.

b. Two of the true statements are logically equivalent. Determine which two. How is the third related to them? (Hint: use the words *converse* and *contrapositive* in your explanation.)

5. Given the set $A = \{1, 2, 3\}$ give
   a. $|A|$
   b. a proper subset of $A$
   c. the power set of $A$

   True or false:
   d. $\{1, 2, 3\} = \{3, 2, 1, 1\}$
   e. $\emptyset \in A$
   f. $\emptyset \subseteq A$
   g. $A \subseteq A$
   h. $1 \in A$
   i. $1 \in A$
   j. $1 \subseteq A$

6. Describe the following set using the set-builder notation: the set of all integers divisible by 3.

7. Let $A = \{x \in Z : 20|x\}$ and $B = \{x \in Z : 10|x\}$. Prove or disprove:
   a. $A \subseteq B$
   b. $B \subseteq A$

8. Consider the statement: $(\exists x \in Z)(\forall y \in Z)(x + y = 0)$.
   a. Translate the statement into plain English.

b. Determine whether the statement is true or false. If false, give a counterexample. If true, explain.

c. Give the negation of the statement, both as a quantified statement and in words.

9.

a. How many different passwords of length 6 can be formed using the 10 digits and 26 letters (such as d83tbk or 4591ja)?

b. How many of them contain at least one digit?

c. How many of them do not contain any character twice in a row? That is, a5a5a5 is OK, but aa38dt is not.