- Suppose for the following data, we construct a Sampling Distribution (i.e., a distribution of means of all possible samples of a given n from a population

\[ M = 100 \]
\[ \sigma = 15 \]
\[ n = 30 \]

- The mean of the sampling distribution would equal \( \mu \). The standard deviation of the sampling distribution (i.e. standard error of the mean) would equal

\[ \sigma_x = \frac{\sigma}{\sqrt{n}} \]

\[ \therefore \quad \mu_x = 100 \]
\[ \sigma_x = \frac{15}{\sqrt{30}} \]
\[ = 2.74 \]

See Figure 1

- Based on what we know about normal distributions:
  a) 1 sd around the mean encompasses \( \sim 68\% \) of the expected means from samples of size \( n=30 \) from the population
  b) 2 sd around the mean encompasses \( \sim 95\% \) of the expected means from samples of size \( n=30 \) from the population

- Based on this example, we would expect the draw a random sample of \( n=30 \) with means \( >105.48 \) or \( <94.52 \) \( 5\% \) of the time

See Figure 2
In a 2-tailed hypothesis test, if we draw a sample that falls in these "tails" (i.e., \( \bar{x} > 105.48 \) or \( \bar{x} < 94.52 \)), we conclude that the probability of drawing such a sample is so unlikely (i.e., < .05) that the sample likely represents a population with a different mean - stated differently - the sample comes from a different population than the hypothesized population.

Of course, we sometimes make a mistake by making such a conclusion because even in the given sampling distribution (Fig. 1) we will occasionally (though rarely) get samples with means that fall within these tails.

In a 1-tailed hypothesis test, we hypothesize that our sample comes from a population with a mean that is either greater than or less than the mean from our sampling distribution.

\[ H_0 : M = 100 \]
\[ H_1 : M > 100 \quad \text{or} \quad M < 100 \]

In this case, we create a "region of rejection" in only 1 tail instead of splitting the 5% between 2 tails.

See Figures 3 and 4.
Fig. 1 Sampling distribution where \( \mu = 100 \) and \( \sigma = 15 \) and \( n = 30 \)

Fig. 2
Fig 3

Sampling Distribution to test the Alternative Hypothesis

$H_1: \mu > 100$

Fig 4

Sampling Distribution to test the hypothesis

$H_1: \mu < 100$
- When doing a 1-tailed test with SPSS, you must determine the appropriate distribution from Figures 3 + 4.

- In SPSS, calculate the 2-tailed probability and divide it by 2.

- The resulting probability will be the 1-tailed probability for your test if your sample mean (\(\bar{x}\)) is on the correct tail for your hypothesis.

A) For example, if my hypotheses were

\[ H_0: \mu = 100 \]
\[ H_1: \mu > 100 \]

and my sample mean was \(\bar{x} = 105\)

b) The probability in SPSS would be .05 (2-tailed)

cut that in half ⇒ .05/2 = .025

c) Because my sample mean (\(\bar{x} = 105\)) is on the right side of my distribution mean (\(\mu = 100\)), I retain the probability of .025 and reject the null.

IF my hypotheses were

\[ H_0: \mu = 100 \]
\[ H_1: \mu < 100 \]

and \(\bar{x} = 105\)

My resulting probability would not be .025 but would be \(1 - .025 = .975\)

and I would fail to reject the null.