A Comparison of Testimation and Schwarz Information Criterion for Heteroscedasticity

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Abstract

In this paper, we introduce the concept of Testimation for detecting heteroscedasticity in the error variability in the context of simple regression. This testimation based method is found to be very useful in detecting heteroscedasticity and even better compared to the Schwarz Information Criterion.

Keywords: Cepheids, *Period-Luminosity Relation, Testimator.*

1. INTRODUCTION

In this paper, we propose an alternative testing method for checking heteroscedasticity among the uncorrelated error terms. We use "testimation" as a procedure to test for heteroscedasticity in the context of simple regression. Previously, the heteroscedasticity was studied by several people in different contexts. Morgan (1939), Pitman (1939), Wilks (1946) were some of the pioneers to investigate heteroscedasticity in the variability of the error terms. Later Theil (1951), Valavanis (1959), Cacoullos (1965), Goldfeldt and Quandt (1965) among many others have studied the heteroscedasticity. In regression, the residuals can be tested for heteroscedasticity by using the tests such as Pitman-Morgan t-test or the Breusch-Pagan test which regresses the squared residuals to the independent variables. However, the Breusch-Pagan test is very sensitive to the error normality and hence a more robust Koenkar-Basset test (or the generalized Breusch-Pagan test) is preferred. For testing group-wise heteroscedasticity, the Goldfeld-Quandt test and the Levene test are commonly used. On the other hand, the concept of testimator(or test-estimator) was first proposed by Bancroft(1944) in the context of estimating a parameter where a prior guess will be used in place of the estimator of an unknown parameter if the prior guess can be ascertained by a test of hypothesis.

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Otherwise the traditional estimator will be used. These testimators are preferable due to their superior efficiency compared to the traditional estimators. This method has been adapted and refined to suit other situations by Paul(1950), Huntsberger (1955), Bancroft (1964), Arnold and Katti (1972), Bock *et al.* (1973), Han (1978), Ghosh and Sinha (1988), Yancey *et al.* (1989), Pandey and Malik (1990), Pandey *et al* (1995), Pandey(1997) and Pandey and Srivastava (2001) to name a few. Waikar *et al.*(1984) and Waikar *et al.*(2001) in work on two-stage shrinkage estimation proposed a weighted testimator by placing a weight k on the prior guess and weight 1-k on the traditional estimator, where k is the probability that the guess can be accurate. They all have shown that the testimators have far superior efficiency and therefore are more powerful. This two stage testimation concept can be refined to test for heteroscedasticity. Recently, Kanbur *et al.* (2009) refined this two-stage Testimation for multiple stages to test for a possible change in the slope of a regression line. In this paper, the Testimation is further refined to test for heteroscedasticity. We describe the methodology in section 2, and in section 4 we present the numerical and graphical results.

2. METHODOLOGY

2.1 Testimator

We develop this Testimation for heteroscedasticity in the error variability in the context of simple regression. In Astrophysics, Hubble's constant determines the size scale of the Universe. Its empirical value is determined by an extra-galactic distance scale whose most important component is based on the properties of Cepheid stars. These are pulsating stars from nearby galaxies with a definite, well defined periodicity. It is found empirically that the mean absolute brightness of Cepheids is linear with respect to the logarithm of their periodicity. Due to this linearity assumption, we fit the model $Y = \alpha + \beta X + e$, where α is the y-intercept, β is the slope, and e is the random measurement error that is independent and normally distributed with mean = 0 and variance = σ_e^2, where σ_e^2 is unknown. This assumption is also supported by the real astrophysical data. The Durbin-Watson test for the Cepheid residual errors based on a linear model show that the error terms are uncorrelated. Also the quantile-quantile plot along with the Shapiro-Wilks test indicated that the residual errors are normally distributed. So, here is how we conduct the Testimation. We partition the entire data into m segments on the basis of the predictor variable which in this case is the logarithm of the Cepheid period on base 10. We divide the data into several non-overlapping intervals (or segments) according to the Cepheid period and then conduct the

Testimation for each segment. Next we conduct the Testimation about the error variability over each interval by using this ''weighted'' or "smoothed" Testimator. Note that the observations in the non-overlapping intervals are independent and therefore the estimates from these non-overlapping intervals are independent (between any two arbitrary segments).

Stage 1:

For the first segment, we formulate the hypothesis as follows.

$H_0 : \sigma_\varepsilon^2 = \sigma_0^2$ versus $H_1 : \sigma_\varepsilon^2 \neq \sigma_0^2$ where σ_0^2 is a guess about the error variance σ_ε^2. Next, we take observations from the first segment to estimate the parameters. First, we estimate the slope. The least-square estimator of the slope β is given by

$$\hat{\beta} = \frac{\sum_1^n \left(x_{1i} - \overline{x}_1\right)\left(y_{1i} - \overline{y}_1\right)}{\sum_1^n \left(x_{1i} - \overline{x}_1\right)^2} \tag{2.1}$$

and the estimate of the y-intercept α is given by

$$\hat{a} = \overline{y}_1 - \hat{\beta}.\overline{x}_1 \tag{2.2}$$

where n is the number of observations sampled and $\overline{x}_1$ and $\overline{y}_1$ are the respective sample means for the X and Y observations in the first segment.

Next, we estimate the error variance estimate as follows.

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{\left(n-2\right)} \sum_1^n \left(y_{1i} - \hat{\alpha} - \hat{\beta} x_{1i}\right)^2 \tag{2.3}$$

and the test-statistic (or evidence) is

$$\chi^2_{obs} = \left(n-2\right)\frac{\hat{\sigma}_\varepsilon^2}{\sigma_0^2} \tag{2.4}$$

Here, we will have two critical values and these are

$$k_1^{(1)} = \frac{\chi^2_{obs}}{\chi^2_{\nu,\alpha/2m}}$$

$$k_2^{(1)} = \frac{\chi^2_{obs}}{\chi^2_{\nu,1-(\alpha/2m)}}$$

with $\nu = n - 2$ for the degrees of freedom.

If $k_1^{(1)} < 1$ and $k_2^{(1)} > 1$ then do not reject H_0. Otherwise reject H_0. (2.5)

Let us define a smoothing constant as follows.

$$k^{(1)} = \frac{k_1^{(1)}}{k_2^{(1)}} \tag{2.6}$$

Note that the Testimation approach is slightly different from the traditional hypothesis testing. Here, while not rejecting the null hypothesis H_0 , we always revise our null value as follows.

$$\sigma^2_{w1} = k^{(1)} \hat{\sigma}_\varepsilon^2 + \left(1 - k^{(1)}\right)\sigma_0^2 \tag{2.7}$$

where $k^{(1)}$ is as defined above.

In multi-stage testing, we repeat this procedure many times.

Stage 2:

Here we will use a new set of n observations from the second segment to test the hypothesis

$$H_0 : \sigma_\varepsilon^2 = \sigma_{w1}^2 \text{ versus } H_1 : \sigma_\varepsilon^2 \neq \sigma_{w1}^2$$

where σ_{w1}^2 is as defined in (2.7) and to proceed with the test, we compute the slope estimate on the second set of n observations.

$$\hat{\beta}^{(2)} = \frac{\sum_1^n \left(x_{2i} - \overline{x}_2\right)\left(y_{2i} - \overline{y}_2\right)}{\sum_1^n \left(x_{2i} - \overline{x}_2\right)^2} \tag{2.8}$$

Note that the y-intercept estimate is $\hat{\alpha}^{(2)} = \overline{y}_2 - \hat{\beta}^{(2)}.\overline{x}_2$ (2.9) where n is the number of observations sampled and $\overline{x}_2$ and $\overline{y}_2$ are the respective sample means for the X and Y observations in the second segment.

Again, we estimate the error variance as follows.

$$\hat{\sigma}_{\varepsilon}^{2^{(2)}} = \frac{1}{\left(n-2\right)} \sum_{1}^{n} \left(y_{2i} - \hat{\sigma}^{2^{(2)}} - \hat{\beta}\ .x_{2i}\right)^{2} \tag{2.10}$$

The test-statistic (evidence) is given by

$$\chi_{obs}{}^{2^{(2)}} = \left(n-2\right) \frac{\hat{\sigma}_{\varepsilon}^{2^{(2)}}}{\sigma_{w1}^{2}} \tag{2.11}$$

Define $$k_1^{(2)} = \frac{\chi_{obs}{}^{2(2)}}{\chi^2_{\nu,\alpha/2m}} \tag{2.12}$$

and $$k_2^{(2)} = \frac{\chi_{obs}{}^{2(2)}}{\chi^2_{\nu,1-(\alpha/2m)}} \tag{2.13}$$

Let $$k^{(2)} = \frac{k_1^{(2)}}{k_2^{(2)}} \tag{2.14}$$

If $k_1^{(2)} < 1$ and $k_2^{(2)} > 1$ then do not reject H_0. Otherwise reject H_0.

Note: The conditional distribution of the test-statistic given by (2.11) given $\sigma_{w1}{}^{2}$ is a chi-square distribution for the reason that the estimates from the first and second segments are independent.

Note that while not rejecting H_0, we will again revise our null value as follows.

$$\sigma^{2}{}_{w2} = k^{(2)} \overset{\wedge}{\sigma}_{\varepsilon}{}^{2(2)} + \left(1 - k^{(2)}\right) \sigma_{w1}{}^{2}$$

We will move to the next stage (Stage 3) and repeat these stage-wise Testimation for all the segments involved to test for a possible change in the error variability.

Next, we will show that the testimator under the null hypothesis H_0 is unbiased about the error variance and has a strictly smaller variance than the usual least square estimator.

Lemma 1: Under H_0 the Testimator is unbiased about the error variance.

Proof: Consider the Testimator in the first stage given by

$$\sigma^2_{w1} = k^{(1)}\hat{\sigma}^2_{\varepsilon} + \left(1 - k^{(1)}\right)\sigma_0^{\,2}$$

where $k^{(1)}$ is as defined earlier.

Note that

$$\begin{aligned} E\left(\sigma^2_{w2}\right) &= k^{(2)} E\left(\hat{\sigma}^{2(2)}_{\varepsilon}\right) + \left(1 - k^{(2)}\right) E\left(\sigma^2_{w1}\right) \\ &= k^{(2)}\sigma_0^{\,2} + \left(1 - k^{(2)}\right)\sigma_0^{\,2} \\ &= \sigma_0^{\,2} \end{aligned}$$

Similarly, the second stage Testimator is given by

$$\cdot\,\sigma^2_{w2} = k^{(2)}\hat{\sigma}^{2(2)}_{\varepsilon} + \left(1 - k^{(2)}\right)\sigma_{w1}^{\,2}$$

and it's expected value

$$\begin{aligned} E\left(\sigma^2_{w2}\right) &= k^{(2)} E\left(\hat{\sigma}^{2(2)}_{\varepsilon}\right) + \left(1 - k^{(2)}\right) E\left(\sigma^2_{w1}\right) \\ &= k^{(2)}\sigma_0^{\,2} + \left(1 - k^{(2)}\right)\sigma_0^{\,2} \\ &= \sigma_0^{\,2} \end{aligned}$$

By repeating these steps, one can easily obtain the following result that at every stage, under H_0

$$E\left(\sigma^2_{w,m}\right) = E\left(\sigma^2_{w,m-1}\right) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = E\left(\sigma^2_{w1}\right) = \sigma_0^{\,2} \qquad (2.15)$$

Moreover in the procedure described above, we use the very conservative Bonferoni multiple testing procedure. The following lemma shows that the probability for committing the Family-wise Error is less than or equal to α.

Lemma 2:

For the Bonferroni type Testimation procedure,

$$P(\text{Family-wise error}) \leq \alpha$$

Proof:

Let A_i = "Making a correct decision in the i^{th} stage"

Then, $\cap A_i$ = "Making the correct decision in all the stages"

Note that by the Multiplication rule,

$$P\left(\cap A_i\right) = P\left(A_1\right).P\left(A_2 \setminus A_1\right).P\left(A_3 \setminus A_2, A_1\right)............P\left(A_m \setminus A_1, A_2,.......A_{m-1}\right)$$

$$= \left(1-\frac{\alpha}{m}\right)\left(1-\frac{\alpha}{m}\right).............\left(1-\frac{\alpha}{m}\right)$$

$$= \left(1-\frac{\alpha}{m}\right)^m \tag{2.16}$$

So, the probability for committing family-wise error rate

P (Family-wise error) = P(At least one mistake in any one of the stages)

$$= 1-\left(1-\frac{\alpha}{m}\right)^m$$

$$\leq \alpha \tag{2.17}$$

Lemma 3: Let 0 < a < 1 and 0 < b < 1 then $a^2 (1-b)^2 + b^2 < 1$.

Proof:

Note that $a^2 (1 - b) < 1 - b < 1 + b$

So, $a^2\,(1 - b)^2 < (1 + b)(1 - b)$

This means, $a^2\,(1 - b)^2 < 1 - b^2$

Hence, $a^2\,(1 - b)^2 + b^2 < 1$

Lemma 4: At every stage ($1 \leq i \leq m$) involved in the Testimation,

$$Var\left(\sigma^2_{wi}\right) < Var\left(\hat{\sigma}^{2(i)}_{\varepsilon}\right)$$

Proof:

Note that $\sigma_{wi}^{\ \ 2} = k^{(i)}\,\hat{\sigma}_{\varepsilon}^{2^{(i)}} + \left(1 - k^{(i)}\right)\sigma^2_{w,i-1}$

$$Var\left(\sigma_{wi}^{\ \ 2}\right) = Var\left(E\left(k^{(i)}\hat{\sigma}_{\varepsilon}^{2(i)}\right) \backslash\, \sigma_{w,i-1}^{\ \ 2}\right) + E\left(\left(1 - k^{(i)}\right)\sigma_{w,i-1}^{\ \ 2} \backslash\, \sigma_{w,i-1}^{\ \ 2}\right) \tag{2.18}$$

$$+E\left(Var\left(k^{(i)}\sigma^2_{w,i-1} + \left(1 - k^{(i)}\right)\sigma_{w,i-1}\right) \backslash\, \sigma_{w,i-1}^{\ \ 2}\right)$$

$$= \left(1 - k^{(i)}\right)^2 k^{(i-1)^2}.2.\frac{\sigma_0^{\ 4}}{(n-1)} + k^{(i)^2}.2.\frac{\sigma_0^{\ 4}}{(n-1)}$$

$$= 2\left(\frac{\sigma_0^{\ 4}}{(n-1)}\right)\left(\left(1 - k^{(i)}\right)^2 k^{(i-1)^2} + k^{(i)^2}\right) \tag{2.19}$$

$$< Var\left(\hat{\sigma}_{\varepsilon}^{2(i)}\right) \tag{2.20}$$

3. SCHWARZ INFORMATION CRITERION

In this section, we present the Schwarz Information Criterion (SIC). It is defined as $-2.\log\left(L\left(\hat{\sigma}\right)\right) + p.\log n$ where p is the number of free parameters and n is the sample size. According to the null hypothesis when there is no change in the variability, we will have only three free parameters.

$$SIC_0 = -2.\log\left(L\left(\hat{\sigma}\right)\right) + 3.\log\left(n\right)$$

$$= n.\log\left(2\pi\right) + n.\log\left(\hat{\sigma}^2\right) + n + 3.\log\left(n\right)$$

Similarly, under the alternative hypothesis when there is a change in the variability, there will be six free parameters.

$$SIC_1\left(k\right) = n.\log\left(2\pi\right) + k.\log\left(\hat{\sigma}_1^2\right) + \left(n-k\right).\log\left(\hat{\sigma}_2^2\right) + n + 6.\log\left(n\right)$$

where σ_1 represents the amount of error variability among the first k observations and σ_2 is the amount of error variability among the last $n - k$ observations.

Note that SIC indicates a change in the error variability when

$$SIC_1(k) < SIC_0 \text{ for some } k.$$

3. The Numerical Results

In all the Tables 1-4, 'segment#' refers to 'segment number'; k_1 and k_2 are as defined in section 2; the column labeled 'change?' checks to see whether there has been a change in the error variability in that particular segment and 'Per' refers to the period range of the segment in question.

First, we present the numerical and graphical results from a simulated astrophysical data set with 1500 observations that is homoscedastic with respect to error variability. This means that there is no change in the error variability. The error terms were simulated in such a way that the errors are independent and follow the normal distribution. Our aim is to compare this Testimation with Schwarz Information Criterion (SIC). We partition the data into 15 segments each with 100 observations. The Testimation based results are listed in Table 1. The SIC based results are given graphically (see Figure 1).

For the following graphs of SIC (Figures 1 through 4),

- Purple graph represents the SIC under the null hypothesis
- Black graph represents the SIC under the alternative hypothesis

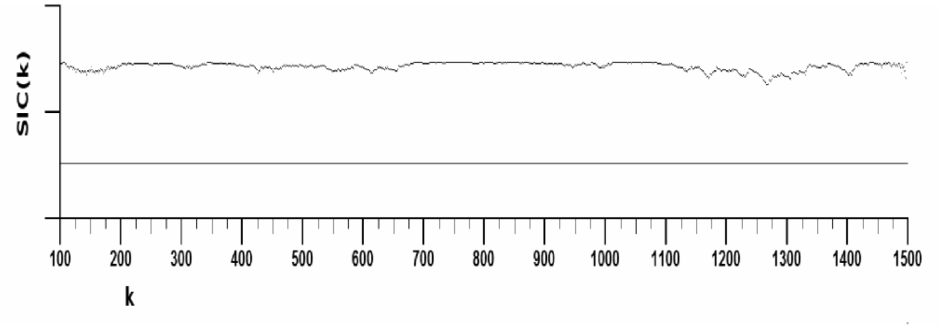

Figure 1. SIC for the simulated data with no variability change.

Table 1. Testimation Results (Simulated data with no variability change)

segment#	k_1	k_2	**change?**	**segment#**	k_1	k_2	**change?**
1	**0.6284**	**1.463**	**no**	**9**	**0.6563**	**1.5282**	**no**
Per. 0.231532-0.39969				*Per. 0.607964-0.650886*			
2	**0.7386**	**1.7197**	**no**	**10**	**0.7692**	**1.791**	**no**
Per. 0.399704-0.442089				*Per. 0.651183-0.734875*			
3	**0.6344**	**1.4771**	**no**	**11**	**0.5137**	**1.1961**	**no**
Per. 0.442215-0.471267				*Per. 0.734915-0.845992*			
4	**0.6857**	**1.5965**	**no**	**12**	**0.6322**	**1.4719**	**no**
Per. 0.471911-0.500512				*Per. 0.847352-0.960996*			
5	**0.7096**	**1.6521**	**no**	**13**	**0.7052**	**1.642**	**no**
Per. 0.500613-0.52474				*Per. 0.961033-1.11802*			
6	**0.6552**	**1.5255**	**no**	**14**	**0.8692**	**2.0238**	**no**
Per. 0.525023-0.550761				*Per. 1.11918-1.3553*			
7	**0.678**	**1.5785**	**no**	**15**	**0.752**	**1.7508**	**no**
Per. 0.551187-0.574372				*Per. 1.36517-2.61702*			
8	**0.7398**	**1.7225**	**no**				
Per. 0.574408-0.607864							

As we can see, Testimation and the SIC lead to the same correct conclusion that there is no change in the error variability among the simulated error terms.

Next, we present the results based on a different simulated data set. Here, the data is heteroscedastic with respect to error variability. In other words, there is a change in the error variability among the simulated error terms. Again we partition the 1500 observations into 15 segments with each segment having 100 observations. Testimation results are listed in Table 2. SIC based results are presented graphically in Figure 2.

Table 2: Testimation Results (Simulated data with variability change)

segment#	k_1	k_2	**change?**	**segment#**	k_1	k_2	**change?**
1	**0.6738**	**1.5689**	**no**	**9**	**0.6568**	**1.5293**	**no**
Per. 0.231532-0.39969				***Per. 0.607964-0.650886***			
2	**0.7692**	**1.7909**	**no**	**10**	**0.7696**	**1.7917**	**no**
Per. 0.399704-0.442089				*Per. 0.651183-0.734875*			
3	**0.6486**	**1.5101**	**no**	**11**	**0.5138**	**1.1964**	**no**
Per. 0.442215-0.471267				*Per. 0.734915-0.845992*			
4	**0.6946**	**1.6173**	**no**	**12**	**0.4518**	**1.052**	**no**
Per. 0.471911-0.500512				*Per. 0.847352-0.960996*			
5	**0.7148**	**1.6643**	**no**	**13**	**0.309**	**0.7195**	**yes**
Per. 0.500613-0.52474				*Per. 0.961033-1.11802*			
6	**0.6579**	**1.5317**	**no**	**14**	**0.8717**	**2.0296**	**no**
Per. 0.525023-0.550761				*Per. 1.11918-1.3553*			
7	**0.6796**	**1.5823**	**no**	**15**	**0.7531**	**1.7533**	**no**
Per. 0.551187-0.574372				*Per. 1.36517-2.61702*			
8	**0.7408**	**1.7249**	**no**				
Per. 0.574408-0.607864							

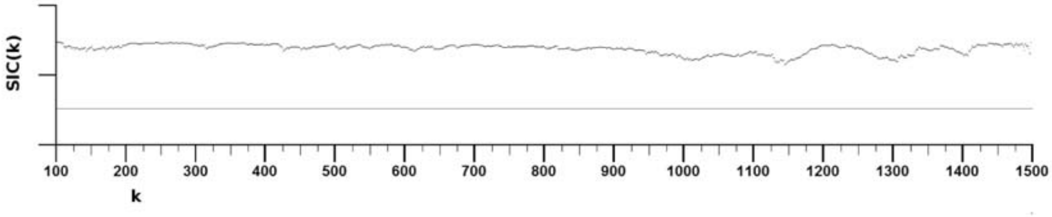

Figure 2. SIC for the simulated data with variability change.

Note that Testimation is detecting the change in error variability in segment 13 whereas SIC is not detecting the change. For SIC to detect a change in the error variability, the "black" graph should go below the "purple" line.

Table 3. Testimation on OGLE III data (I- Band) (Real Data)

segment#	k_1	k_2	**change?**	**segment#**	k_1	k_2	**change?**
1	**0.6256**	**1.4729**	**no**	**10**	**0.7645**	**1.8002**	**no**
Per. 0.014863-0.362307				*Per. 0.574069-0.606824*			
2	**0.7066**	**1.6637**	**no**	**11**	**0.6928**	**1.6312**	**no**
Per. 0.363113-0.419294				*Per. 0.606929-0.636973*			
3	**0.6201**	**1.46**	**no**	**12**	**0.5988**	**1.41**	**no**
Per. 0.419871-0.446623				*Per. 0.637351-0.675302*			
4	**0.7305**	**1.72**	**no**	**13**	**0.9792**	**2.3056**	**no**
Per. 0.446959-0.469515				*Per. 0.675848-0.719402*			
5	**0.75**	**1.766**	**no**	**14**	**0.9249**	**2.1777**	**no**
Per. 0.469638-0.488356				*Per. 0.721337-0.805033*			
6	**0.5019**	**1.1819**	**no**	**15**	**0.6729**	**1.5844**	**no**
Per. 0.488442-0.505856				*Per. 0.805521-0.915883*			
7	**0.5389**	**1.2688**	**no**	**16**	**0.6185**	**1.4563**	**no**
Per. 0.505939-0.526188				*Per. 0.916875-1.12331*			
8	**0.7835**	**1.8448**	**no**	**17**	**0.5039**	**1.7538**	**no**
Per. 0.526312-0.548222				*Per. 1.12711-1.49207*			
9	**0.6452**	**1.5192**	**no**				
Per. 0.548676-0.574044							

Next, we analyze real astrophysical data from the OGLE III project in the I-band with more than 1600 observations. We partitioned the data into 17 segments with the first 16 having 100 observations each and the remainder in the 17th. Testimation results are in Table 3. SIC results are in Figure 3.

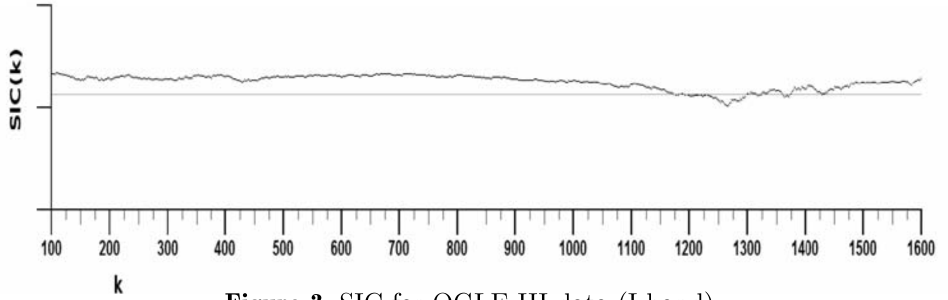

Figure 3. SIC for OGLE III data (I-band)

Table 4. Testimation for OGLE III data (V- Band) (Real Data)

segment#	k_1	k_2	**change?**	**segment#**	k_1	k_2	**change?**
1	**0.5195**	**1.2353**	**no**	**11**	**0.7914**	**1.8819**	**no**
Per. 0.014863-0.354671				*Per. 0.58285-0.614067*			
2	**0.8347**	**1.9848**	**no**	**12**	**0.8547**	**2.0325**	**no**
Per. 0.356353-0.414647				*Per. 0.614405-0.644675*			
3	**0.4241**	**1.0084**	**no**	**13**	**0.5077**	**1.2074**	**no**
Per. 0.415314-0.443928				*Per. 0.644783-0.679712*			
4	**0.547**	**1.3009**	**no**	**14**	**0.4934**	**1.1734**	**no**
Per. 0.444172-0.464946				*Per. 0.679881-0.72133*			
5	**0.9479**	**2.2542**	**no**	**15**	**1.1942**	**2.8398**	**yes**
Per. 0.464954-0.482567				*Per. 0.721866-0.794294*			
6	**0.5274**	**1.2542**	**no**	**16**	**1.1054**	**2.6287**	**yes**
Per. 0.482572-0.497792				*Per. 0.794479-0.869169*			
7	**0.4468**	**1.0626**	**no**	**17**	**1.0497**	**2.4961**	**yes**
Per. 0.497998-0.515763				*Per. 0.869998-1.01315*			
8	**0.7975**	**1.8964**	**no**	**18**	**0.6451**	**1.5341**	**no**
Per. 0.515886-0.536666				*Per. 1.01485-1.34864*			
9	**1.3669**	**3.2504**	**yes**	**19**	**0.5833**	**3.255**	**no**
Per. 0.536726-0.557498				*Per. 1.35302-1.71911*			
10	**0.5485**	**1.3043**	**no**				
Per. 0.558254-0.582348							

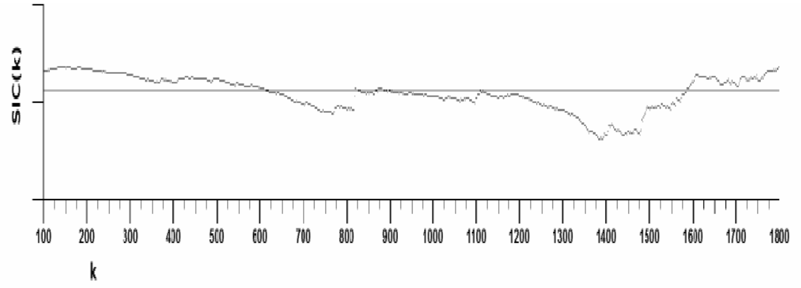

Figure 4. SIC for OGLE III data (V band)

Again, we analyze real astrophysical data from the OGLE III project in the V band with more than 1800 observations. We partitioned the data into 19 segments with the first 18 having 100 observations each and the remainder in the 19th. Testimation results are in Table 4. SIC results are in Figure 4.

4. CONCLUSION

We have developed a method based on testimation to detect heteroscedasticity in linear regression data. We have tested this method and compared it with the Schwarz Information Criterion (SIC). For simulated data where we can infuse heteroscedasticity at a known location, and we have found that the testimator method correctly identifies the presence of heteroscedasticity in the error variability and its location. However, the SIC fails in this respect. For the real astronomical data (V-Band), we find both SIC and Testimator methods provide strong statistical evidence to support the occurrence of heteroscedasticity. For the V-band data, both methods find a change in variance of the data at periods of about 10 days and periods of about 2.5 days. At both these periods, there is strong evidence from astrophysical work to suggest possible causes behind this heteroscedasticity (Kanbur and Ngeow, 2004; Kanbur and Ngeow 2010). Both methods do not find any change in the error variance in the I-band – this is to be expected on astrophysical grounds since the amplitudes of Cepheids in the I-band is smaller than in the V-band. Moreover compared to the other tests that are available in the literature, the advantage with the Testimator approach is that it is structured to control the family-wise error rate for making misjudgment at any stage. Furthermore, by incorporating smoothing at every stage, the Testimator also removes the bad influence due to the outliers. This is important in the Astrophysical context where an individual data point could have a large error due to sudden changes in the atmosphere. In addition, the Testimator estimates have smaller variability than the unbiased estimates for the error variance. Also note that Testimation is a stepwise procedure and it permits constraints on the period ranges in which heteroscedasticity could occur with respect to error variability. Again this is important in the Astrophysical context because a certain amount of scatter in the PL relation is due to physical causes.

In Astrophysics, the heteroscedasticity in the context of Cepheid-PL relation could mean the following:

(1) The region defined by the instability strip in the Hertzsprung-Russell diagram where these Cepheids are located could have a variable width.

(2) Some internal physics is taking place which is causing stars in certain period ranges to oscillate with either greater or reduced variability. This is why a stepwise procedure like Testimation is preferred over other existing methods.

Moreover, it is well-known in Astrophysics, the Hertzsprung-Russell progression is centered on a period of 10 days. According to our investigation, the error variability change also occurs at periods of about 10 days. This is a new finding and of great interest in Astrophysics.

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