Chapter 26

The Refraction of Light: Lenses and Optical Instruments
Light travels through a vacuum at a speed $c = 3.00 \times 10^8 \text{ m/s}$

Light travels through materials at a speed less than its speed in a vacuum.

**DEFINITION OF THE INDEX OF REFRACTION**

The index of refraction of a material is the ratio of the speed of light in a vacuum to the speed of light in the material:

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in the material}} = \frac{c}{v}$$
## 26.1 The Index of Refraction

<table>
<thead>
<tr>
<th>Substance</th>
<th>Index of Refraction, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solids at 20 °C</strong></td>
<td></td>
</tr>
<tr>
<td>Diamond</td>
<td>2.419</td>
</tr>
<tr>
<td>Glass, crown</td>
<td>1.523</td>
</tr>
<tr>
<td>Ice (0 °C)</td>
<td>1.309</td>
</tr>
<tr>
<td>Sodium chloride</td>
<td>1.544</td>
</tr>
<tr>
<td>Quartz</td>
<td></td>
</tr>
<tr>
<td>Crystalline</td>
<td>1.544</td>
</tr>
<tr>
<td>Fused</td>
<td>1.458</td>
</tr>
<tr>
<td><strong>Liquids at 20 °C</strong></td>
<td></td>
</tr>
<tr>
<td>Benzene</td>
<td>1.501</td>
</tr>
<tr>
<td>Carbon disulfide</td>
<td>1.632</td>
</tr>
<tr>
<td>Carbon tetrachloride</td>
<td>1.461</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>1.362</td>
</tr>
<tr>
<td>Water</td>
<td>1.333</td>
</tr>
<tr>
<td><strong>Gases at 0 °C, 1 atm</strong></td>
<td></td>
</tr>
<tr>
<td>Air</td>
<td>1.000 293</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>1.000 45</td>
</tr>
<tr>
<td>Oxygen, $O_2$</td>
<td>1.000 271</td>
</tr>
<tr>
<td>Hydrogen, $H_2$</td>
<td>1.000 139</td>
</tr>
</tbody>
</table>

*a Measured with light whose wavelength in a vacuum is 589 nm.*
26.2 Snell’s Law and the Refraction of Light

SNELL’S LAW

When light travels from a material with one index of refraction to a material with a different index of refraction, the angle of incidence is related to the angle of refraction by

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
Example 1 Determining the Angle of Refraction

A light ray strikes an air/water surface at an angle of 46 degrees with respect to the normal. Find the angle of refraction when the direction of the ray is (a) from air to water and (b) from water to air.
26.2 Snell’s Law and the Refraction of Light

\[ \sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} = \frac{1.00 \sin 46^\circ}{1.33} = 0.54 \]

\[ \theta_2 = 33^\circ \]

(b) \[ \sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} = \frac{1.33 \sin 46^\circ}{1.00} = 0.96 \]

\[ \theta_2 = 74^\circ \]
Example 2  Finding a Sunken Chest

The searchlight on a yacht is being used to illuminate a sunken chest. At what angle of incidence should the light be aimed?
26.2 Snell’s Law and the Refraction of Light

\[
\sin \theta_1 = \frac{n_2 \sin \theta_2}{n_1} = \frac{1.33 \sin 31^\circ}{1.00} = 0.69
\]

\[
\theta_1 = 44^\circ
\]

\[
\theta_2 = \tan^{-1}\left(\frac{2.0}{3.3}\right) = 31^\circ
\]
26.2 Snell’s Law and the Refraction of Light

Apparent depth, observer directly above object

\[ d' = d \left( \frac{n_2}{n_1} \right) \]
Conceptual Example 4  On the Inside Looking Out

A swimmer is under water and looking up at the surface. Someone holds a coin in the air, directly above the swimmer’s eyes. To the swimmer, the coin appears to be at a certain height above the water. Is the apparent height of the coin greater, less than, or the same as its actual height?
26.2 Snell’s Law and the Refraction of Light

THE DISPLACEMENT OF LIGHT BY A SLAB OF MATERIAL
26.2 Snell’s Law and the Refraction of Light

THE DERIVATION OF SNEILL’S LAW

(a) Incident ray

(b) Refracted ray

Fast-moving incident wave fronts

Slow-moving refracted wave fronts
26.3 *Total Internal Reflection*

When light passes from a medium of larger refractive index into one of smaller refractive index, the refracted ray bends away from the normal.

![Diagram of light passing through different mediums showing critical angle and total internal reflection.](image)

**Critical angle**

\[
\sin \theta_c = \frac{n_2}{n_1} \quad n_1 > n_2
\]
Example 5  Total Internal Reflection

A beam of light is propagating through diamond and strikes the diamond-air interface at an angle of incidence of 28 degrees. (a) Will part of the beam enter the air or will there be total internal reflection? (b) Repeat part (a) assuming that the diamond is surrounded by water.
### 26.3 Total Internal Reflection

(a) \[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) = \sin^{-1} \left( \frac{1.00}{2.42} \right) = 24.4^\circ \]

(b) \[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) = \sin^{-1} \left( \frac{1.33}{2.42} \right) = 33.3^\circ \]
Conceptual Example 6 The Sparkle of a Diamond

The diamond is famous for its sparkle because the light coming from it glitters as the diamond is moved about. Why does a diamond exhibit such brilliance? Why does it lose much of its brilliance when placed under water?
26.3 *Total Internal Reflection*
26.3 Total Internal Reflection

(a) Light ray
(b) Cladding
(c) Core
(d) Air
(e) Core
(f) Cladding

\[ \theta_1, \theta_2, \theta_c \]
Brewster's law

\[ \tan \theta_B = \frac{n_2}{n_1} \]
The net effect of a prism is to change the direction of a light ray. Light rays corresponding to different colors bend by different amounts.
26.5 The Dispersion of Light: Prisms and Rainbows

Table 26.2  Indices of Refraction $n$ of Crown Glass at Various Wavelengths

<table>
<thead>
<tr>
<th>Approximate Color</th>
<th>Wavelength in Vacuum (nm)</th>
<th>Index of Refraction, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>660</td>
<td>1.520</td>
</tr>
<tr>
<td>Orange</td>
<td>610</td>
<td>1.522</td>
</tr>
<tr>
<td>Yellow</td>
<td>580</td>
<td>1.523</td>
</tr>
<tr>
<td>Green</td>
<td>550</td>
<td>1.526</td>
</tr>
<tr>
<td>Blue</td>
<td>470</td>
<td>1.531</td>
</tr>
<tr>
<td>Violet</td>
<td>410</td>
<td>1.538</td>
</tr>
</tbody>
</table>
Conceptual Example 7  The Refraction of Light Depends on Two Refractive Indices

It is possible for a prism to bend light upward, downward, or not at all. How can the situations depicted in the figure arise?
26.5 *The Dispersion of Light: Prisms and Rainbows*
Lenses refract light in such a way that an image of the light source is formed.

With a converging lens, paraxial rays that are parallel to the principal axis converge to the focal point.
With a diverging lens, paraxial rays that are parallel to the principal axis appear to originate from the focal point.
26.6 *Lenses*

Converging lenses
- Double convex
- Plano-convex
- Convex meniscus

Diverging lenses
- Double concave
- Plano-concave
- Concave meniscus
26.7 The Formation of Images by Lenses

RAY DIAGRAMS
In this example, when the object is placed further than twice the focal length from the lens, the real image is inverted and smaller than the object.
When the object is placed between F and 2F, the real image is inverted and larger than the object.
When the object is placed between F and the lens, the virtual image is upright and larger than the object.
A diverging lens always forms an upright, virtual, diminished image.
The Thin-Lens Equation and the Magnification Equation

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

\[
m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}
\]
Summary of Sign Conventions for Lenses

\[ f \text{ is } + \text{ for a converging lens.} \]
\[ f \text{ is } - \text{ for a diverging lens.} \]
\[ d_o \text{ is } + \text{ if the object is to the left of the lens.} \]
\[ d_o \text{ is } - \text{ if the object is to the right of the lens.} \]
\[ d_i \text{ is } + \text{ for an image formed to the right of the lens (real image).} \]
\[ d_i \text{ is } - \text{ for an image formed to the left of the lens (virtual image).} \]
\[ m \text{ is } + \text{ for an upright image.} \]
\[ m \text{ is } - \text{ for an inverted image.} \]
**Example 9 The Real Image Formed by a Camera Lens**

A 1.70-m tall person is standing 2.50 m in front of a camera. The camera uses a converging lens whose focal length is 0.0500 m. (e) Find the image distance and determine whether the image is real or virtual. (b) Find the magnification and height of the image on the film.

(a) \[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{0.0500 \text{ m}} - \frac{1}{2.50 \text{ m}} = 19.6 \text{ m}^{-1}
\]

\[
d_i = 0.0510 \text{ m} \quad \text{real image}
\]

(b) \[
m = -\frac{d_i}{d_o} = -\frac{0.0510 \text{ m}}{2.50 \text{ m}} = -0.0204
\]

\[
h_i = mh_o = \left(-0.0204\right)\left(2.50 \text{ m}\right) = -0.0347 \text{ m}
\]
The image produced by one lens serves as the object for the next lens.
26.10 The Human Eye

ANATOMY

- Aqueous humor
- Iris
- Cornea
- Lens
- Suspensory ligament
- Ciliary muscle
- Retina
- Vitreous humor
- Optic nerve
The lens only contributes about 20-25% of the refraction, but its function is important.
The lens creates an image of the distance object at the far point of the nearsighted eye.
Example 12 Eyeglasses for the Nearsighted Person

A nearsighted person has a far point located only 521 cm from the eye. Assuming that eyeglasses are to be worn 2 cm in front of the eye, find the focal length needed for the diverging lens of the glasses so the person can see distant objects.
26.10 The Human Eye

\[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} - \frac{1}{519 \text{ cm}} \]

\[ f = -519 \text{ cm} \]
The lens creates an image of the close object at the near point of the farsighted eye.
Optometrists who prescribe correctional lenses and the opticians who make the lenses do not specify the focal length. Instead they use the concept of refractive power.

\[
\text{Refractive power (in diopters)} = \frac{1}{f \text{ (in meters)}}
\]
The size of the image on the retina determines how large an object appears to be.
26.11 Angular Magnification and the Magnifying Glass

\[
\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}}
\]

\[
\theta \; \text{(in radians)} \approx \frac{h_o}{d_o}
\]
Example 14  A Penny and the Moon

Compare the angular size of a penny held at arms length with that of the moon.

Penny

\[
\theta \approx \frac{h_o}{d_o} = \frac{1.9 \text{ cm}}{71 \text{ cm}} = 0.027 \text{ rad}
\]

Moon

\[
\theta \approx \frac{h_o}{d_o} = \frac{3.5 \times 10^6 \text{ m}}{3.9 \times 10^8 \text{ m}} = 0.0090 \text{ rad}
\]
Angular Magnification and the Magnifying Glass

Angular magnification

\[ M = \frac{\theta'}{\theta} \]

Angular magnification of a magnifying glass

\[ M \approx \left( \frac{1}{f} - \frac{1}{d_i} \right) N \]
To increase the angular magnification beyond that possible with a magnifying glass, an additional converging lens can be included to “premagnify” the object.

$$M \approx -\frac{(L - f_e)N}{f_o f_e}$$
Angular magnification of an astronomical telescope

\[ M \approx -\frac{f_o}{f_e} \]
In a converging lens, spherical aberration prevents light rays parallel to the principal axis from converging at a single point.

Spherical aberration can be reduced by using a variable-aperture diaphragm.
Chromatic aberration arises when different colors are focused at different points along the principal axis.