Lewis, the lottery and the preface

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David Lewis (1999) sketches a contextualist account of our epistemic situation in the cases of lotteries. That story generates a new puzzle, one that he doesn’t address. Solving the puzzle requires a subtle emendment of the theory, one that in turn sheds new light on the paradox of the preface.

1. The theory

Context-dependence is commonly believed to be a widespread phenomenon in natural language. In a context where the standards of precision are very demanding (say, a rocket launch), an utterance of ‘It’s 3 pm’ said at 3.01 will be reckoned false. Reduce the standards of precision and an utterance of the same sentence at the same time will be reckoned true. Semanticists habitually accommodate phenomena of this type by allowing that utterances of the same sentence type express different propositions on different occasions of use, owing to variations along contextual parameters that are not explicitly marked in the surface structure of the sentence itself.

Contextualists in epistemology hold that the verb ‘know’ provides yet another case of context-dependence, expressing diverse relations on different occasions of use owing to shifting standards for its application. In Lewis’s particular version of contextualism, the key contextual parameter is the divide between the possible worlds that are being properly ignored by the ascriber and those that are not. He offers the following definition:

S knows proposition P iff S’s evidence eliminates every possibility in which not-P – Psst! – except for those possibilities that we are properly ignoring. (Lewis 1999: 425)

Here’s the idea: A speaker makes a knowledge ascription to a subject, deploying a claim of the form ‘S knows that P’. Call the possibilities that the speaker is not properly ignoring the ‘relevant possibilities’. The ascription is true just in case there is no relevant possibility where the subject’s evidence is just as it actually is, but where P is not true. Relevance, note, is determined by what the speaker is properly ignoring, not what the subject is properly ignoring. Within this framework, it is easy enough to see what the semantic value of ‘know’ will be on a particular occasion of use: on a given occasion, that verb will pick out a relation R between subjects and propositions, such that a subject stands in R to a proposition P iff there are...
no possibilities that are relevant to the ascriber on that occasion which are such that the subject’s evidence is as it actually is and not-\(P\) obtains.

What makes it true or false on some particular occasion that a possibility is relevant? Lewis (1999: 426ff.) offers a set of guidelines, four of which will be especially relevant to us. First, the Rule of Actuality: the actual world can never be properly ignored. Second, the Rule of Attention: if a possibility is being attended to by an ascriber, it is automatically relevant. (Note that the fact that a subject is attending to a possibility need not make it relevant.) Third, the Rule of Resemblance: if a possibility saliently resembles a relevant possibility, then both possibilities are relevant. Fourth, the Rule of Belief: that a possibility that is believed by the subject to obtain – or which is a possibility that ought to be believed by the subject to obtain – is relevant. An alternative gloss: if a possibility is given sufficiently high credence by the subject – or ought to be – then it is relevant.

What is a possibility? We cannot think of a possibility as a single world, else we could never attend to a possibility – except perhaps the actual world – since our cognitive mechanisms are not so discriminating as to allow us to lock on to a single world. We cannot either think of a possibility as any class of worlds – any Lewisian proposition – since in that case, whenever we are attending to the content of a question of the form ‘Tell me whether or not \(P\)’, we would be attending to a possibility in which not-\(P\) (which will nearly always contain at least some subcases where my evidence matches the actual world). We would then be threatened with the result that the very asking of a question of that form places one in a sceptical context with regard to \(P\) (except in the special case where our evidence is strictly incompossible with the proposition that not-\(P\)). Lewis strikes a via media:

A possibility will be specific enough if it cannot be split into subcases in such a way that anything we have said about possibilities, or anything we are going to say before we are done, applies to some subcases and not to others. (Lewis 1999: 426)

The proposition that there is a dog in front of me is not, then, a possibility in the relevant sense, since it enjoys readily articulable subcases whose difference certainly matters when it comes to epistemology (say, as between my hallucinating a dog and there happening to be one anyway as opposed to a dog producing a visual representation as of it).

2. Lotteries

When, in advance of a lottery drawing, and without special insider information, I announce, ‘I do not know that I will lose the lottery,’ then, whether or not I end up winning or losing, my assertion expresses a truth. Lewis deploys the resources of his theory to explain why:
It is the Rule of Resemblance that explains why you do not know that you will lose the lottery, no matter what the odds are against you and no matter how sure you should therefore be that you will lose. For every ticket, there is the possibility that it will win. These possibilities are saliently similar to one another: so either every one of them may be properly ignored, or else none may. But one of them may not be properly ignored: the one that actually obtains. (Lewis 1999: 430)

Lotteries present a puzzle to non-sceptics. If I do not know that I will win a lottery, I do not know either, say, whether I will be someone who has a surprise fatal heart attack, or whether I will be the unlucky victim of theft and so on. But if I do not know such things, then apparently humdrum knowledge of the future – that I will be teaching next week, that I will be driving to work in my own car, and so on – is threatened.¹

One way to handle the threat is to deny closure. I know that I will be at work next week. That entails I will not have a fatal heart attack before. But I do not know the latter: nor am I in a position to. Whatever the merits of such a tack, it is out of the question for Lewis. He is operating within an epistemic framework that obliterates hyperintensional distinctions. If P and Q are true in exactly the same set of possible worlds, then one knows P if and only if one knows Q. This is easily seen from his definition: suppose that necessarily, P iff Q and there are no relevant possibilities where (i) not-P and (ii) one’s evidence is as it actually is. Then there are no relevant possibilities where not-Q and one’s evidence is as it actually is.² Lewis’s definition also directly secures the thesis that if P entails Q and one knows P, then one knows Q, and also the thesis that if one knows P and one knows Q, then one knows P and Q. For if, in a given case, there are no relevant not-P possibilities and P entails Q, then there are no relevant not-Q possibilities. (If there were a relevant possibility in which not-Q, it would have to be a relevant possibility in which not-P.) Similarly, for any given P and Q, if there are no relevant not-P worlds and no relevant not-Q worlds, then there are no relevant not-(P and Q) worlds. It is no surprise, then, that Lewis announces that his is a ‘modal’ epistemol-

¹ Jonathan Vogel’s work is especially good on this. See, for example Vogel 1990. The generality of the problem is insufficiently appreciated in a recent extended treatment of the lottery problem in Nelkin 2000.

² Notice that, assuming (i) that knowledge distributes over conjunction, and (ii) that knowledge that P entails knowledge of any Q that is modally equivalent to Q, we can directly deduce that if P entails Q, knowledge that P entails knowledge that Q: if I know that I will be in school next week, I know by (i) the conjunction (I will be in school next week and I will not have a fatal heart attack beforehand). By (ii), I know that I will not have a fatal heart attack beforehand.
ology, where the axioms common to normal modal logics govern the knowl-
edge operator.³

Here is Lewis’s contextualism-driven response to the sceptical threat, one
that is consistent with modal epistemology:

Pity poor Bill! He squanders all his spare cash on the pokies, the races,
and the lottery. He will be a wage slave all his days. We know he will
never be rich. But if he wins the lottery (if he wins big), then he will be
rich. … So, by closure, if we know that he will never be rich, we know
that he will lose. But when we discussed the case before, we concluded
that we cannot know that he will lose … But there is a loophole: the
resemblance was required to be salient. Salience, as well as ignoring,
may vary between contexts. Before, when I was explaining how the
Rule of Resemblance applies to lotteries, I saw to it that the resem-
blance between the many possibilities associated with the many tickets
was sufficiently salient. But this time, when we were busy pitying poor
Bill for his habits and not for his luck, the resemblance of the many
possibilities was not so salient. At that point the possibility of Bill’s
winning was properly ignored; so then it was true to say that we knew
he would never be rich. … It was true at first that we knew that Bill
would never be rich. And at that point it was also true that we knew
he would lose – but that was true so long as it remained unsaid! …
Later, after the change in context, it was no longer true that we knew
he would lose. At that point, it was also no longer true that we knew

The solution easily generalizes. Suppose Bill says of Ben, ‘I know Ben will
be in work on Monday’, properly ignoring the possibility that Ben might
be one of the unlucky people to have a fatal heart attack before Monday.
In Bill’s mouth, the verb ‘know’ expresses a relation that holds between
himself and the proposition that Ben will be in work on Monday and
also (by closure) between himself and the proposition that Ben will not
have a fatal heart attack before Monday. (And this is so even if Ben is
attending to the possibility of a heart attack, since the Rule of Attention
applies to the ascriber, not the subject.) If Bill goes on to attend to the pos-
sibility⁴ of Ben’s having a fatal heart attack before Monday, he will have
put himself in a context where the verb ‘know’, if deployed by him,

³ Thus the distribution principle that \(K(P \supset Q) \supset KQ\) entails \(KP \supset KQ\), as well as the prin-
ciple that all logical truths are known, is assumed by the framework. (The latter
principle, note, also flows from Lewis’s anti-hyperintensionalism.) Notice that the
distribution law, in combination with classical logic, can be used to derive the closure
principles mentioned in the body of the text, as well as the principle that knowledge
distributes over conjunction.

⁴ Of course, strictly speaking, there are many such possibilities.
would express a different relation, one that does not hold between Bill and the proposition that Ben will be in work on Monday, nor between Bill and the proposition that Ben will not have a heart attack before Monday.

3. A puzzling case

Ben has many friends. They are all ‘losers’. Poor Bill squanders his money on the pokies, the races, and the lottery. So does poor Harry. So does poor Gerald. So does poor William … Ben has five thousand such friends. Each buys a lottery ticket. Only one non-friend buys a lottery ticket, who later turns out to be the winner. (There are 5,001 tickets.) Ben is very familiar with the lifestyle of his friends and is also well aware how the 5,001 lottery tickets were distributed. An ascriber looks on and says, ‘Ben knows poor Bill will never get rich.’ The ascriber, we may suppose, is not attending to such possibilities as that Bill will win the lottery. That possibility is properly ignored by the ascriber.

Lewis tells us what to say: given the context, the ascription expresses a truth. With the possibility of surprise lottery success still firmly out of view, the ascriber goes on: ‘What’s more, Ben knows poor Harry will never get rich.’ Lewis tells us what to say: given the context, the ascription expresses a truth.

The ascriber goes on: ‘Further, Ben knows poor Gerald will never get rich. He’s another loser.’ And so on. The ascriber picks on each of Ben’s loser friends and makes a similar speech.5 By Lewis’s lights, each speech expresses a truth.

But now we appear to have a problem. Call the relation expressed by the verb ‘know’ in the context of the ascriber ‘K’. We seem to have it that Ben K’s that Bill will never get rich, and that Ben K’s that Harry will never get rich, and so on. By closure, Ben K’s that none of his 5,000 friends will get rich. By closure Ben K’s that none of his 5,000 friends will win the lottery. And as if that were not bad enough, it is pretty clear that, given the non-sceptical standards for knowledge-ascription being deployed by the ascriber, Ben K’s that there are 5,001 lottery tickets, 5,000 of which are owned by his friends. So, by closure, Ben K’s that the single non-friend to hold a lottery ticket will win the lottery. Of course, the relevant proposition has to remain unsaid by the ascriber. If the ascriber were later to utter ‘Ben knows that his non-friend will win the lottery’, he would

5 If you like, imagine that S has a few friends with very promising futures – Harold and Maude – and suppose that the point of the ascriber’s speech is to contrast S’s epistemic situation with regard to the prospects of his loser friends with his situation regarding the prospects of Harold and Maude. So at the end, the ascriber says ‘On the other hand, S does not know whether or not Harold and Maude will be rich.’
have changed the context in such a way that ‘know’ would no longer express K. It would express another relation K2 that does not hold between Ben and the proposition that his non-friend will win. But isn’t it bad enough that the theory has it that the verb ‘know’ sometimes expresses a relation that holds between a person and the proposition that some individual will win the lottery, in advance of a drawing and without privileged information?6

To fix your intuitions further. Suppose Ben, as a joke, or as a misleading attempt to communicate information he does not have, says, out of earshot of the ascriber, ‘My non-friend will win the lottery.’ The ascriber mistakenly thinks that Ben has just said something else, say ‘Bill is my friend.’ The ascriber says, ‘Ben knows that what he just said was true.’ According to the current result, the ascriber has, by luck, expressed a truth, even though what Ben just said was ‘My non-friend will win.’

4. A solution

There is little room for manoeuvre. Lewis is committed to closure. And he is committed to a picture according to which, so long as the ascriber does not attend to lottery-winning possibilities and the like, each knowledge ascription expresses a truth in his mouth. Nor is it plausible that we can dissolve the puzzle by insisting that, by the ascriber’s standards, Ben fails to know that there are 5,001 lottery tickets, 5,000 of which are owned by his friends. Ben may have even seen his friends buy up 5,000 of the lottery tickets one day and have a memory trace of that event. Sceptical worlds in which such events are hallucinated are being properly ignored by the ascriber, since the ascriber is, presumably, in a context where such wild possibilities are irrelevant.

As a preliminary towards a solution, let us recall the Rule of Belief, according to which a possibility that is accorded sufficiently high credence by the subject – or ought to be accorded high credence – cannot be properly ignored. Now the subject accords high credence – or if he doesn’t he ought to – to the proposition that one of his friends will win the lottery. Now that proposition is not a possibility in Lewis’s sense – it is not specific enough. But the following is, surely, a plausible principle:

New Rule of Belief: If the proposition that P is given sufficiently high credence – or ought to be – by the subject, then one cannot properly ignore all of the possibilities that constitute subcases of P.

6 And leaving aside the special case where, say, the name ‘Bill’ is introduced by the reference-fixing description, ‘Let “Bill” name the lottery winner’, enabling one to go on to say, ‘Bill will win the lottery.’ I shall not enquire here how to handle such cases within the framework of a theory that obliterates hyperintensional distinctions.
The subject accords high credence to the proposition that one of his friends will win. This proposition divides into various subcases – some in which Bill wins, some in which Gerald wins, and so on. By the New Rule of Belief, not all of these subcases can be properly ignored. Suppose that an ascriber is properly ignoring all the possibilities in which, say, Bill wins the lottery. He cannot, by our principle, also be properly ignoring all the possibilities in which one of the other friends win. In short, whether or not they are actually attended to by the ascriber, some friend-winning possibilities must always be counted as relevant to knowledge ascriptions to Ben, since Ben accords high credence to the proposition that one of his friends will win.

With this in mind, a solution is ready to hand. When the puzzle was framed, we assumed that there was a single context in which the ascriber’s various speech acts were performed and, relatedly, a single relation K that was expressed by each deployment of the verb ‘know’. This assumption has to be dropped. When the ascriber says ‘Ben knows that Bill will never get rich’, he is properly ignoring the possibility that Bill will win the lottery. But is he on that occasion properly ignoring the possibility that Gerald will win the lottery? The New Rule of Belief says that at least some friend-winning possibilities will have to be relevant. We may as well say that, on that occasion, all the possibilities involving a friend other that Bill winning were relevant, consistently maintaining that the possibility of Bill winning was properly ignored. (For on what basis can one exempt Gerald but not, say, William?) One might worry that the Rule of Resemblance now gets one in trouble. The possibility of Gerald winning is, by hypothesis, not properly ignored when the speech about Bill is made. But doesn’t the possibility of Bill’s winning resemble the possibility of Gerald winning? And doesn’t this mean that, by the Rule of Resemblance, either both possibilities are relevant or neither are? The worry is groundless. It is crucial to the Rule of Resemblance that it operate on salient resemblances. (Recall Lewis’s earlier discussion.) It is clear enough, in the context of the speech about Bill, that the resemblance between the relevant possibility of Gerald winning and the possibility of Bill winning is not salient.

The speech continues. ‘Ben knows that Gerald will never get rich.’ In this setting the possibility that Gerald will win the lottery is now properly ignored. But by the New Rule of Belief, some friend-winning possibilities must be relevant. Perhaps the possibility that Bill will win is not relevant yet – the preconditions of the truth of the earlier speech act can be allowed to remain in force for a short while. But what is clear is that, sooner or later, the possibility that Bill will win must become relevant in order that the last speech act of the form ‘Ben knows that S will never get rich’ be true in the mouth of the ascriber. Given the extended belief principle, some possibilities whose irrelevance was crucial to the truth of earlier speech acts in our
sequence must become relevant by the time of the last speech act in our sequence in order for that last speech act to be true. Assuming that we make the necessary accommodations to allow that each speech act be true (adhering, *ceteris paribus*, to the rules of accommodation that prevail over our language game), we can be sure that some possibilities swing from being irrelevant to relevant. Accordingly, we can be sure that the semantic value of ‘know’ is far from being constant during the conversation. There is no relation $K$, expressed by some use of the verb ‘know’ such that Ben stands in relation $K$ to each proposition of the form ‘$S$ will never get rich’, where $S$ is identical to a friend.

5. A version of the preface

The standard version of the paradox of the preface is as follows: an author believes each of the claims in a long manuscript and yet also believes that one of the claims is false, expressing the latter belief in the preface to a book. (See Makinson 1965.) It might well be reasonable for the author to believe the contents of the manuscript and also reasonable for her to believe that it contains a false claim. Yet these beliefs are contradictory. We appear to have a counter-example to the initially compelling principle that it is irrational to hold contradictory beliefs.

This is not a puzzle about knowledge. Knowledge is factive (hence the Rule of Actuality). It is clear from the outset that not every claim made in the manuscript, together with the relevant claim from the preface, can be known to be true. Nevertheless, it is not hard to construct a preface-style paradox for knowledge. Consider:

I write down a manuscript consisting of a very long sequence of sentences, $s_1^s$, sincerely saying to myself at each juncture, ‘I know that what I’m writing down is true.’ Let us label these silent soliloquies $k_1^s$. Let us

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7 Consideration of the version of the preface that follows should forestall any inclination on the part of the reader to insist that, given the earlier speech acts, there is no way that the final one can express a truth, on the grounds that the ascriber traps himself into not properly ignoring the possibility of lottery victory by the final friend (the earlier conversation, in effect, requiring that all the other friends’ possible victories remain properly ignored throughout the conversation, the New Rule of Belief requiring that some friend’s possible victory not be properly ignored). That would be like saying that, having got up in the morning and said, knowledgeably, on inductive grounds, ‘I’ll probably say something false sometime today’, this automatically means that if I know what I said to be true earlier in the day, I cannot know what I say late at night.

8 I have in mind Lewis 1983. We should expect that Lewis adopt a rule of accommodation for proper ignoring, viz: if at time $t$ something is said that requires that possibility $P$ be properly ignored to be acceptable, then – *ceteris paribus* and within certain limits – possibility $P$ is properly ignored. (Cf. Lewis 1983: 234.)
presume that each sentence has, individually, high epistemic credentials, and that, as it turns out, each sentence is true. Let us suppose, indeed, that by ordinary standards each individual sentence would be reckoned to express a piece of knowledge. Before or after running through the sequence, I say, on inductive grounds, ‘It is very likely that I will make (will have just made) a mistake. After all, every manuscript that any of my highly intelligent friends has ever written has contained a mistake.’ Once again, by ordinary standards, this would appear to express a piece of knowledge. Even if, as chance would have it, the sequence $s^1$ to $s^n$ is mistake free, the claim about likelihood would appear to be both true and known. Assuming closure, it seems that I both know that a long conjunction is true and also that it is likely to be false.

What is going on? Let us address this from the perspective of Lewisian contextualism. Suppose each of $k^1$ to $k^n$ self-ascribes a single knowledge relation $K$ between the subject (who in this case is also the ascriber) and the propositions $p^1$–$p^n$ expressed by $s^1$–$s^n$, and that each self-ascription is correct. By closure, the subject $K$’s the conjunction of $s^1$–$s^n$. Assuming that the subject also $K$’s that each of his highly intelligent friends has made a mistake whenever writing a manuscript, we can infer, by closure, that the subject $K$’s that he alone, among all his friends, has succeeded in writing a mistake-free manuscript. A fantastic result, I think, not much less fantastic that the idea that ‘know’ might express a relation that Ben stands to the proposition that his non-friend will win the lottery. The air of paradox is not resolved simply by saying that the prefatory speech is made in a different context, one where another knowledge relation $K_2$ is in play. For that so-called solution concedes that in the context of $k^1$–$k^n$, the verb ‘know’ expresses a relation that the subject holds to the conjunction of the propositions expressed by $s^1$–$s^n$, and, indeed, to the proposition that he is unique among his friends.

A solution is ready to hand, one that mimicks our solution to the earlier puzzle. I have high credence that, when I write down a long sequence, I will make a mistake. By the New Rule of Belief, not every possibility in which a member of the long sequence is mistaken can be properly ignored. Certainly, when I write down $s^1$, I may properly ignore possibilities in which the proposition expressed by $s^1$ is false and my evidence is as it actually is (assuming that $s^1$ has, individually, high epistemic credentials). But, when I give soliloquy $k^1$, I cannot in that context be properly ignoring all the possibilities where another member of the sequence is mistaken. As the sequence $k^1$–$k^n$ progresses, it is clear enough what we should say: the verb ‘know’ shifts in semantic value. Accordingly, there is no single relation $K$ that is self-ascribed to each proposition in the soliloquy, and thus no relation $K$ that is, by closure, self-ascribed to the conjunction and in turn to the proposition that I, alone among my friends, has succeeded in writing a
mistake-free manuscript. The apparent tension of $k^1-k^n$ with the epistemic credentials of my prefatory remarks thus evaporates, and this without abandoning a commitment to closure in its strongest form. The knowledge-theoretic version of the paradox of the preface disappears.9

6. Concluding remarks

The solution to our lottery puzzle required that we posit rapid context shifting where, initially, context shift was far from noticeable. Within the framework of Lewis’s theory, this result is, I have argued, inevitable. He needs the result to avoid unpalatable commitments. And, in any case, the result is close to mandated by the combination of his own remarks and an extremely plausible emendment to the Rule of Belief. With these elements in place, we found ourselves with new resources to call upon when confronted with a knowledge-theoretic version of the preface paradox. Some readers, admittedly, will have lost patience well before our dissolution of that paradox. The kind of hyperactive context shifting required by our refined theory will seem utterly anathema to them. Such readers will believe that our lottery puzzle makes vivid the radical implausibility of Lewis’s theory, and any theory relevantly like it. Another kind of reader will reckon the set of solutions to the companion puzzles rather satisfying. I shall not adjudicate here.

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References


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