

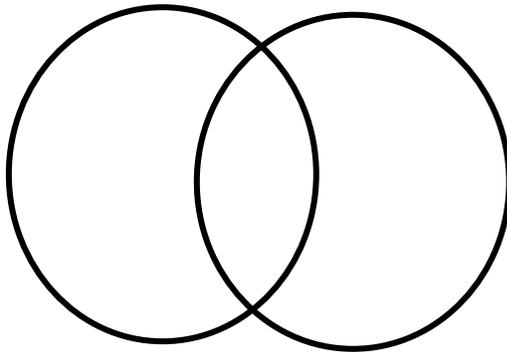
Homework: Theory Concepts and Godel Numbering

I. Theory Concepts

Use the following figure to illustrate the concepts about our theories. Assume we are talking about a theory T . For Venn diagrams, we put an “X” in an area to say, *there is at least one thing in this area*; and we shade an area to say, *there is nothing in this area*. Alter the following diagrams to represent the kind of situation described below the diagram. Remember that what goes into the circles are various sentences that we can write using our theory T .

Sentences that are provable in T .

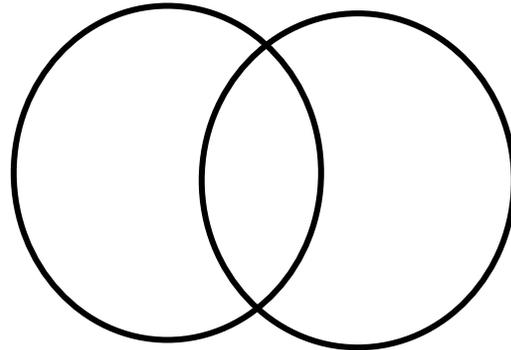
Sentences that are true in T .



T is complete

Provable in T

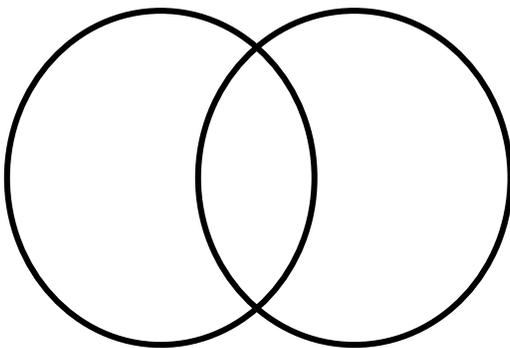
True in T



T is consistent

Provable in T

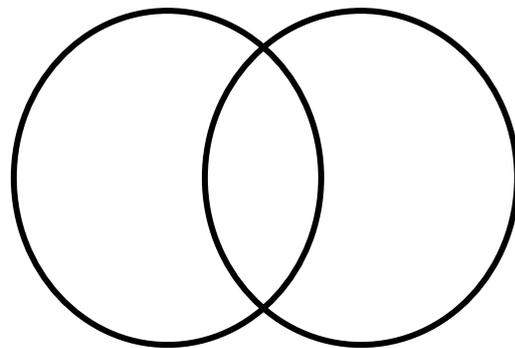
True in T



T is complete but inconsistent

Provable in T

True in T



T is incomplete but consistent

II. Godel Numbering

Our Godel numbering will use the following pattern:

Elements:	'	0	()	f	*	v	~	→	∀	=	≤	#
Numbering:	0	1	2	3	4	5	6	7	8	9	a	b	c

And we will define “f*” to mean the same thing as our “+”. A number n is “0” followed by n of “(””. If we add two numbers to make a new number, we must put this function expression in parenthesis: for example, for the expression “one plus zero,” we write “(1+0)” in normal mathematical writing, and in our logical system this is written “(0' f* 0)”. For this homework, you’ll need only the first six elements of this logical language.

Another example: the expression, “two plus two is four” is in our everyday writing done like this:

$$(2+2) = 4$$

And in our logical system is written:

$$(0' f* 0'') = 0''''$$

And this will have the Godel number:

2100451003a10000

You can see all kinds of things from this. For example: “1” followed by zeros is a number equal to the quantity of zeros; “45” means plus; whatever appears between “2” and “3” is nested in parenthesis; etc.

Let’s call an expression a *question* if it is of the form of an unfinished equality or inequality. So, this is a question: “(2+2)=”. Which would be written in our logical system as “(0' f* 0'') =” and would have the Godel number **2100451003a**. And, this is a question: “(2+2)≤”, with Godel number **2100451003b**.

Your homework is to do four things for each of the following questions: a. tell us what it says in our normal way of writing math; b. tell us the answer in our normal way of writing; c. write the whole equation or inequality in our logical notation; and then d. give the Godel number for the whole equation or inequality. (Thus, using the example above: I give you the question “**2100451003a**”. Your answers are: **a.** (2+2)= **b.** 4. **c.** (0' f* 0'') = 0'''' and **d.** **2100451003a10000**.) Here are the questions:

- (1) 210004510003a
- (2) 2100000004513a
- (3) 21045103a
- (4) 22104510034521000451000033a
- (5) 100000a
- (6) 2100000004513b
- (7) 22104510034521000451000033b
- (8) 100000b