## 8B: $3^l$, $2^k \cdot x^\alpha$, $y^\beta$, $6 \cdot z^\gamma$

**x = 5, y = 7, z = 11:** $2^k \cdot 5^\alpha = 7^\beta - 1 = 6 \cdot (7^{\beta-1} + ... + 1)$ is divisible by 3, contradiction.

**x = 5, y = 11, z = 7:** Both $3^l$ and $3^l - 3 = 6 \cdot 7^\gamma - 6 = 6 \cdot (7^\gamma - 1) = 6 \cdot 6 \cdot (7^{\gamma-1} + ... + 1)$ are divisible by 9, contradiction. (We may and do assume $l \geq 2$.)

**x = 7, y = 5, z = 11:** l must be odd, otherwise both $3^l + 1 = 2^k \cdot 7^\alpha$ and $3^l - 1 = 3^{2l'} - 1 = 9^{l'} - 1 = 8 \cdot (9^{l'-1} + ... + 1)$ would be divisible by 4. It follows that $2^k \cdot 7^\alpha = 3^l + 1 = 4 \cdot (3^{l-1} - 3^{l-2} + ... + 1)$ with $3^{l-1} - 3^{l-2} + ... + 1$ odd, therefore $k = 2$. Now $\gamma$ must be even, otherwise $4 \cdot 7^\alpha + 8 = 6 \cdot 11^\gamma + 6 = 6 \cdot (11^\gamma + 1) = 6 \cdot 12 \cdot (11^{\gamma-1} - 11^{\gamma-2} + ... + 1)$ would be divisible by 8. But with $\gamma$ even both $3^l$ and $3^l - 3 = 6 \cdot 11^\gamma - 6 = 6 \cdot (11^{2\gamma'} - 1) = 6 \cdot (121^{\gamma'} - 1) = 6 \cdot 120 \cdot (121^{\gamma'-1} + ... + 1)$ would be divisible by 9, contradiction. (We may and do assume $l \geq 2$.)

**x = 7, y = 11, z = 5:** $2^k \cdot 7^\alpha = 11^\beta - 1 = 10 \cdot (11^{\beta-1} + ... + 1)$ is divisible by 5, contradiction.

**x = 11, y = 5, z = 7:** As in case z = 7 above.

**x = 11, y = 7, z = 5:** $2^k \cdot 11^\alpha = 7^\beta - 1 = 6 \cdot (7^{\beta-1} + ... + 1)$ is divisible by 3, contradiction.