

7B: $2^k \cdot 3$, x^α , $2 \cdot y^\beta$, $3^l \cdot z^\gamma$

$x = 5, y = 7, z = 11$: α must be even, otherwise $2 \cdot 7^\beta = 5^\alpha + 1 = 6 \cdot (5^{\alpha-1} - 5^{\alpha-2} + \dots + 1)$ would be divisible by 3. In fact $\alpha' = \alpha/2$ must be even as well, otherwise $2 \cdot 7^\beta = 5^\alpha + 1 = 5^{2\alpha'} + 1 = 25^{\alpha'} + 1 = 26 \cdot (25^{\alpha'-1} - 25^{\alpha'-2} + \dots + 1)$ would be divisible by 13. But then $2^k \cdot 3 = 5^\alpha - 1 = 5^{4\alpha''} - 1 = 625^{\alpha''} - 1 = 624 \cdot (625^{\alpha''-1} + \dots + 1)$ would be divisible by 13, contradiction.

$x = 5, y = 11, z = 7$: $2^k \cdot 3 = 2 \cdot 11^\beta - 2 = 2 \cdot (11^\beta - 1) = 2 \cdot 10 \cdot (11^{\beta-1} + \dots + 1)$ is divisible by 5, contradiction.

$x = 7, y = 5, z = 11$: α must be even, otherwise $2 \cdot 5^\beta = 7^\alpha + 1 = 8 \cdot (7^{\alpha-1} - 7^{\alpha-2} + \dots + 1)$ would be divisible by 2^3 . In fact $\alpha' = \alpha/2$ must be even as well, otherwise $2^k \cdot 3 = 7^\alpha - 1 = 7^{2\alpha'} - 1 = 49^{\alpha'} - 1 = 48 \cdot (49^{\alpha'-1} + \dots + 1)$ leads to $2^{k-4} = 49^{\alpha'-1} + \dots + 1$ with $49^{\alpha'-1} + \dots + 1$ odd: this might happen only for $k = 4$ and $\alpha' = 1$, leading to $3^l \cdot 11^\gamma = 51$. But then $2^k \cdot 3 = 7^\alpha - 1 = 7^{4\alpha''} - 1 = 2,401^{\alpha''} - 1 = 2,400 \cdot (2,401^{\alpha''-1} + \dots + 1)$ would be divisible by 5, contradiction.

$x = 7, y = 11, z = 5$: exactly as in case $y = 11$ above.

$x = 11, y = 5, z = 7$: $2^k \cdot 3 = 11^\alpha - 1 = 10 \cdot (11^{\alpha-1} + \dots + 1)$ is divisible by 5, contradiction.

$x = 11, y = 7, z = 5$: exactly as in case $x = 11$ above.