

4B: $2 \cdot x^\alpha, y^\beta, 2^k \cdot 3^l, z^\gamma$

$x = 5, y = 7, z = 11$: $2^k \cdot 3^l = 11^\gamma - 1 = 10 \cdot (11^{\gamma-1} + \dots + 1)$ is divisible by 5, contradiction.

$x = 5, y = 11, z = 7$: β must be odd, otherwise $2 \cdot 5^\alpha = 11^\beta - 1 = 11^{2\beta'} - 1 = 121^{\beta'} - 1 = 120 \cdot (121^{\beta'-1} + \dots + 1)$ would be divisible by 3. It follows that $2^k \cdot 3^l = 11^\beta + 1 = 12 \cdot (11^{\beta-1} - 11^{\beta-2} + \dots + 1)$ with $11^{\beta-1} - 11^{\beta-2} + \dots + 1$ odd, therefore $k = 2$. Now γ must be odd, otherwise $4 \cdot 3^l = 7^\gamma - 1 = 7^{2\gamma'} - 1 = 49^{\gamma'} - 1 = 48 \cdot (49^{\gamma'-1} + \dots + 1)$ would be divisible by 2^4 . But if γ is odd then $4 \cdot 3^l = 7^\gamma - 1 = 6 \cdot (7^{\gamma-1} + \dots + 1)$ with $7^{\gamma-1} + \dots + 1$ odd, contradiction.

$x = 7, y = 5, z = 11$: as in case $z = 11$ above or in case $y = 5$ below.

$x = 7, y = 11, z = 5$: $2 \cdot 7^\alpha = 11^\beta - 1 = 10 \cdot (11^{\beta-1} + \dots + 1)$ is divisible by 5, contradiction.

$x = 11, y = 5, z = 7$: $2 \cdot 11^\alpha = 5^\beta - 1 = 4 \cdot (5^{\beta-1} + \dots + 1)$ is divisible by 2^2 , contradiction.

$x = 11, y = 7, z = 5$: $2 \cdot 11^\alpha = 7^\beta - 1 = 6 \cdot (7^{\beta-1} + \dots + 1)$ is divisible by 3, contradiction.