3B: $2^k \cdot x^\alpha$, y^β, $2\cdot 3^l$, z^γ

<u>**x = 5, y = 7, z = 11:**</u> $2^k \cdot 5^\alpha = 7^\beta - 1 = 6\cdot(7^{\beta-1} + ... + 1)$ is divisible by 3, contradiction.

<u>**x = 5, y = 11, z = 7:**</u> β must be even, otherwise $2\cdot 3^l = 11^\beta + 1 = 12\cdot(11^{\beta-1} - 11^{\beta-2} + ... + 1)$ would be divisible by 2^2. But then $2^k \cdot 5^\alpha = 11^\beta - 1 = 11^{2\beta'} - 1 = 121^{\beta'} - 1 = 120\cdot(121^{\beta'-1} + ... + 1)$ would be divisible by 3, contradiction.

<u>**x = 7, y = 5, z = 11:**</u> $2\cdot 3^l = 11^\gamma - 1 = 10\cdot(11^{\gamma-1} + ... + 1)$ is divisible by 5, contradiction.

<u>**x = 7, y = 11, z = 5:**</u> $2\cdot 3^l = 5^\gamma - 1 = 4\cdot(5^{\gamma-1} + ... + 1)$ is divisible by 2^2, contradiction.

<u>**x = 11, y = 5, z = 7:**</u> β must be odd, otherwise $2^k \cdot 11^\alpha = 5^\beta - 1 = 5^{2\beta'} - 1 = 25^{\beta'} - 1 = 24\cdot(25^{\beta'-1} + ... + 1)$ would be divisible by 3. But if β is odd then $2\cdot 3^l = 5^\beta + 1 = 6\cdot(5^{\beta-1} - 5^{\beta-2} + ... + 1)$, $3^{l-1} = (2\cdot 3 - 1)^{\beta-1} - (2\cdot 3 - 1)^{\beta-2} + ... + 1$, and, assuming $l \geq 2$, 3 divides $(-1)^{\beta-1} - (-1)^{\beta-2} + ... + 1 = \beta$. It follows that $2^k \cdot 11^\alpha = 5^\beta - 1 = 5^{3\beta''} - 1 = 125^{\beta''} - 1 = 124\cdot(125^{\beta''-1} + ... + 1)$ is divisible by 31, contradiction.

<u>**x = 11, y = 7, z = 5:**</u> $2^k \cdot 11^\alpha = 7^\beta - 1 = 6\cdot(7^{\beta-1} + ... + 1)$ is divisible by 3, contradiction.