

2B: $2 \cdot x^\alpha, 3^l, 2^k \cdot y^\beta, z^\gamma$

$x = 5, y = 7, z = 11$: $2^k \cdot 7^\beta = 11^\gamma - 1 = 10 \cdot (11^{\gamma-1} + \dots + 1)$ is divisible by 5, contradiction.

$x = 5, y = 11, z = 7$: $2^k \cdot 11^\beta = 7^\gamma - 1 = 6 \cdot (7^{\gamma-1} + \dots + 1)$ is divisible by 3, contradiction.

$x = 7, y = 5, z = 11$: γ must be even, otherwise $2^k \cdot 5^\beta = 11^\gamma - 1 = 10 \cdot (11^{\gamma-1} + \dots + 1)$ with $11^{\gamma-1} + \dots + 1$ odd would be impossible in view of $k \geq 2$. But then $2^k \cdot 5^\beta = 11^\gamma - 1 = 11^{2\gamma} - 1 = 121^\gamma - 1 = 120 \cdot (121^{\gamma-1} + \dots + 1)$ would be divisible by 3, contradiction.

$x = 7, y = 11, z = 5$: γ must be odd, otherwise $2^k \cdot 11^\beta = 5^\gamma - 1 = 5^{2\gamma} - 1 = 25^\gamma - 1 = 24 \cdot (25^{\gamma-1} + \dots + 1)$ would be divisible by 3. It follows that $2^k \cdot 11^\beta = 5^\gamma - 1 = 4 \cdot (5^{\gamma-1} + \dots + 1)$ with $5^{\gamma-1} + \dots + 1$ odd, therefore $k = 2$. Now α must be even, otherwise $4 \cdot 11^\beta = 2 \cdot 7^\alpha + 2 = 2 \cdot (7^\alpha + 1) = 2 \cdot 8 \cdot (7^{\alpha-1} - 7^{\alpha-2} + \dots + 1)$ would be divisible by 2^4 . Setting $\alpha = 2\alpha'$, we see that α' must also be even, otherwise $4 \cdot 11^\beta = 2 \cdot 7^{2\alpha'} + 2 = 2 \cdot (49^{\alpha'} + 1) = 2 \cdot 50 \cdot (49^{\alpha'-1} - 49^{\alpha'-2} + \dots + 1)$ would be divisible by 5. But then both 5^γ and $5^\gamma - 5 = 2 \cdot 7^\alpha - 2 = 2 \cdot (7^{4\alpha''} - 1) = 2 \cdot (2,401^{\alpha''} - 1) = 2 \cdot 2,400 \cdot (2,401^{\alpha''-1} + \dots + 1)$ would be divisible by 5^2 , contradiction. (We may and do assume $\gamma \geq 2$.)

$x = 11, y = 5, z = 7$: $2^k \cdot 5^\beta = 7^\gamma - 1 = 6 \cdot (7^{\gamma-1} + \dots + 1)$ is divisible by 3, contradiction.

x = 11, y = 7, z = 5: Arguing as in case $z = 5$ above we see that γ must be odd, and, via $2^k \cdot 7^\beta = 4 \cdot (5^{\gamma-1} + \dots + 1)$, that $k = 2$. It follows that α is even, otherwise $2^k \cdot 7^\beta = 2 \cdot 11^\alpha + 2 = 2 \cdot (11^\alpha + 1) = 2 \cdot 12 \cdot (11^{\alpha-1} - 11^{\alpha-2} + \dots + 1)$ would be divisible by 3. We conclude that β must be even as well, otherwise both $2 \cdot 11^\alpha - 2 = 2 \cdot (11^{2\alpha'} - 1) = 2 \cdot (121^{\alpha'} - 1) = 2 \cdot 120 \cdot (121^{\alpha'-1} + \dots + 1)$ and $2 \cdot 11^\alpha + 6 = 4 \cdot 7^\beta + 4 = 4 \cdot (7^\beta + 1) = 4 \cdot 8 \cdot (7^{\beta-1} - 7^{\beta-2} + \dots + 1)$ would be divisible by 16. Setting $\beta = 2\beta'$ and assuming $\gamma \geq 2$ we see that $5^\gamma = 4 \cdot 7^{2\beta'} + 1 = 4 \cdot 49^{\beta'} + 1 = 4 \cdot (2 \cdot 5^2 - 1)^{\beta'} + 1 = 25 \cdot M + 4 \cdot (-1)^{\beta'} + 1$ means that either -3 is divisible by 25 (in case β' is odd) or that 5 is divisible by 25 (in case β' is even): either way we have arrived at a contradiction.