

2A: $x^\alpha, 2^k \cdot y^\beta, 3^l, 2 \cdot z^\gamma$

$x = 5, y = 7, z = 11$: $2^k \cdot 7^\beta = 2 \cdot 11^\gamma - 2 = 2 \cdot (11^\gamma - 1) = 2 \cdot 10 \cdot (11^{\gamma-1} + \dots + 1)$ is divisible by 5, contradiction.

$x = 5, y = 11, z = 7$: $2^k \cdot 11^\beta = 2 \cdot 7^\gamma - 2 = 2 \cdot (7^\gamma - 1) = 2 \cdot 6 \cdot (7^{\gamma-1} + \dots + 1)$ is divisible by 3, contradiction.

$x = 7, y = 5, z = 11$: $2 \cdot 11^\gamma - 1 = 3^l$ implies that 3 divides $2 \cdot (12 - 1)^\gamma - 1$ and $2 \cdot (-1)^\gamma - 1$, therefore γ is odd. Now $2^k \cdot 5^\beta = 2 \cdot 11^\gamma - 2 = 2 \cdot (11^\gamma - 1) = 2 \cdot 10 \cdot (11^{\gamma-1} + \dots + 1)$ with $11^{\gamma-1} + \dots + 1$ odd yields $k = 2$. It follows that α is even, otherwise $4 \cdot 5^\beta = 7^\alpha + 1 = 8 \cdot (7^{\alpha-1} - 7^{\alpha-2} + \dots + 1)$. But then both $7^\alpha + 1 = 2^k \cdot 5^\beta$ and $7^\alpha - 1 = 7^{2\alpha'} - 1 = 49^{\alpha'} - 1 = 48 \cdot (49^{\alpha'-1} + \dots + 1)$ would be divisible by 4, contradiction.

$x = 7, y = 11, z = 5$: $3^l + 1 = 2 \cdot 5^\gamma$ is divisible by 10, so 3^l ends in a 9, which happens precisely when $l = 4l' + 2$. Now $2 \cdot 5^\gamma = 3^l + 1 = 3^{4l'+2} + 1 = 9^{2l'+1} + 1 = 10 \cdot (9^{2l'} - 9^{2l'-1} + \dots + 1)$, so that $5^{\gamma-1} = (2 \cdot 5 - 1)^{2l'} - (2 \cdot 5 - 1)^{2l'-1} + \dots + 1$. We may and do assume $\gamma \geq 2$, so that 5 divides $(-1)^{2l'} - (-1)^{2l'-1} + \dots + 1 = 2l' + 1$ and $l = 2 \cdot (2l' + 1)$ is divisible by 10. But then $2^k \cdot 11^\beta = 3^l - 1 = 3^{10l''} - 1 = (3^{10})^{l''} - 1 = (3^{10} - 1) \cdot ((3^{10})^{l''-1} + \dots + 1)$ would be divisible by $2^3 \cdot 11^2 \cdot 61$, contradiction.

$x = 11, y = 5, z = 7$: $2^k \cdot 5^\beta = 2 \cdot 7^\gamma - 2 = 2 \cdot (7^\gamma - 1) = 2 \cdot 6 \cdot (7^{\gamma-1} + \dots + 1)$ is divisible by 3, contradiction.

$x = 11, y = 7, z = 5$: α must be even, otherwise $2^k \cdot 7^\beta$

$= 11^\alpha + 1 = 12 \cdot (11^{\alpha-1} - 11^{\alpha-2} + \dots + 1)$ would be divisible by 3. But then both $11^\alpha + 1 = 2^k \cdot 7^\beta$ and $11^\alpha - 1 = 11^{2\alpha'} - 1 = 121^{\alpha'} - 1 = 120 \cdot (121^{\alpha'-1} + \dots + 1)$ would be divisible by 4, contradiction.