

# 1A: $x^\alpha, 2 \cdot y^\beta, 3^l, 2^k \cdot z^\gamma$

$x = 5, y = 7, z = 11$ :  $3^l - 1 = 2 \cdot 7^\beta$  yields  $2 \cdot (3^{l-1} + \dots + 1) = 2 \cdot 7^\beta$  and  $3^{l-1} + \dots + 1 = 7^\beta$ , so  $l$  must be odd. With  $l$  odd,  $2^k \cdot 11^\gamma = 3^l + 1 = 4 \cdot (3^{l-1} - 3^{l-2} + \dots + 1)$ ; since the second factor is odd, we conclude that  $k = 2$ . But in that case  $5^\alpha - 1 = 2^k \cdot 11^\gamma - 4 = 4 \cdot (11^\gamma - 1) = 4 \cdot 10 \cdot (11^{\gamma-1} + \dots + 1)$  would be divisible by 5, contradiction.

$x = 5, y = 11, z = 7$ :  $5^\alpha - 1 = 2 \cdot 11^\beta - 2 = 2 \cdot (11^\beta - 1) = 2 \cdot 10 \cdot (11^{\beta-1} + \dots + 1)$  is divisible by 5, contradiction.

$x = 7, y = 5, z = 11$ :  $3^l - 1 = 2 \cdot 5^\beta$  yields  $2 \cdot (3^{l-1} + \dots + 1) = 2 \cdot 5^\beta$  and  $3^{l-1} + \dots + 1 = 5^\beta$ , so  $l$  must be odd. Arguing as in the case  $z = 11$  case above, we conclude that  $k = 2$  and that  $2 \cdot 5^\beta - 2 = 2^k \cdot 11^\gamma - 4$  is divisible by 5, contradiction.

$x = 7, y = 11, z = 5$ :  $2^k \cdot 5^\gamma - 4 = 2 \cdot 11^\beta - 2 = 2 \cdot (11^\beta - 1) = 2 \cdot 10 \cdot (11^{\beta-1} + \dots + 1)$  is divisible by 5, contradiction.

$x = 11, y = 5, z = 7$ :  $2 \cdot 5^\beta - 2 = 11^\alpha - 1 = 10 \cdot (11^{\alpha-1} + \dots + 1)$  is divisible by 5, contradiction.

$x = 11, y = 7, z = 5$ :  $2^k \cdot 5^\gamma - 4 = 11^\alpha - 1 = 10 \cdot (11^{\alpha-1} + \dots + 1)$  is divisible by 5, contradiction.