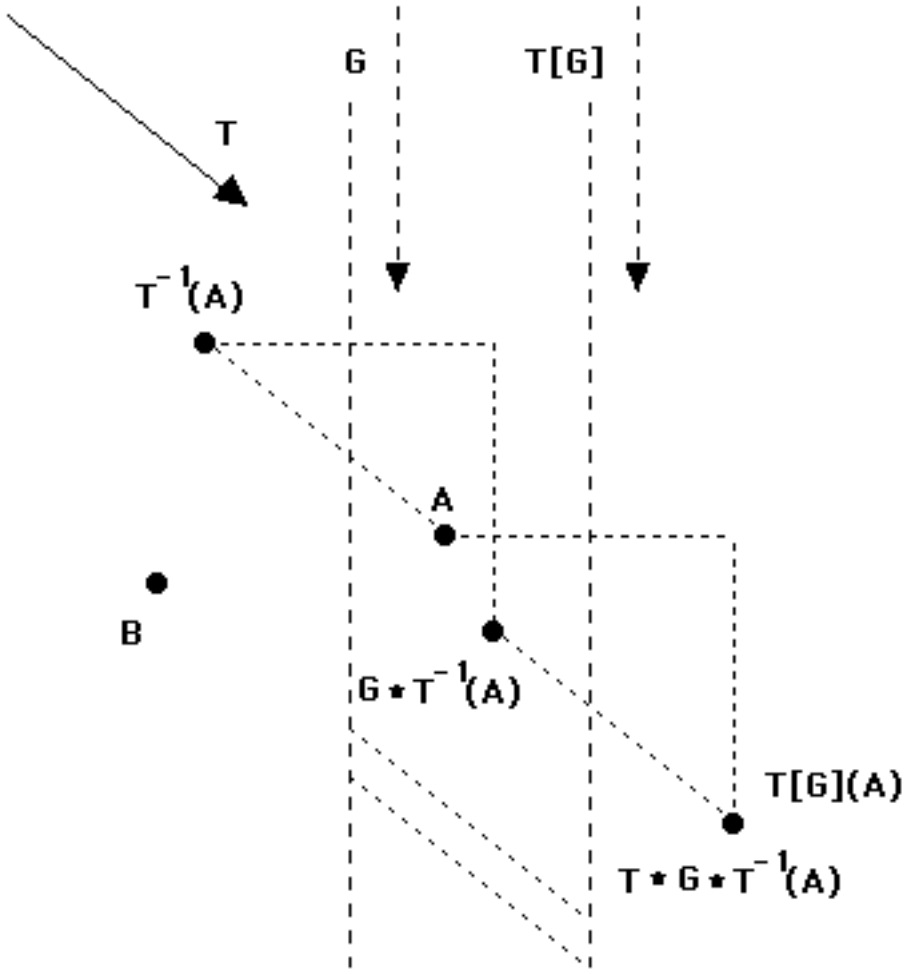
MAT 203 -- Homework #2

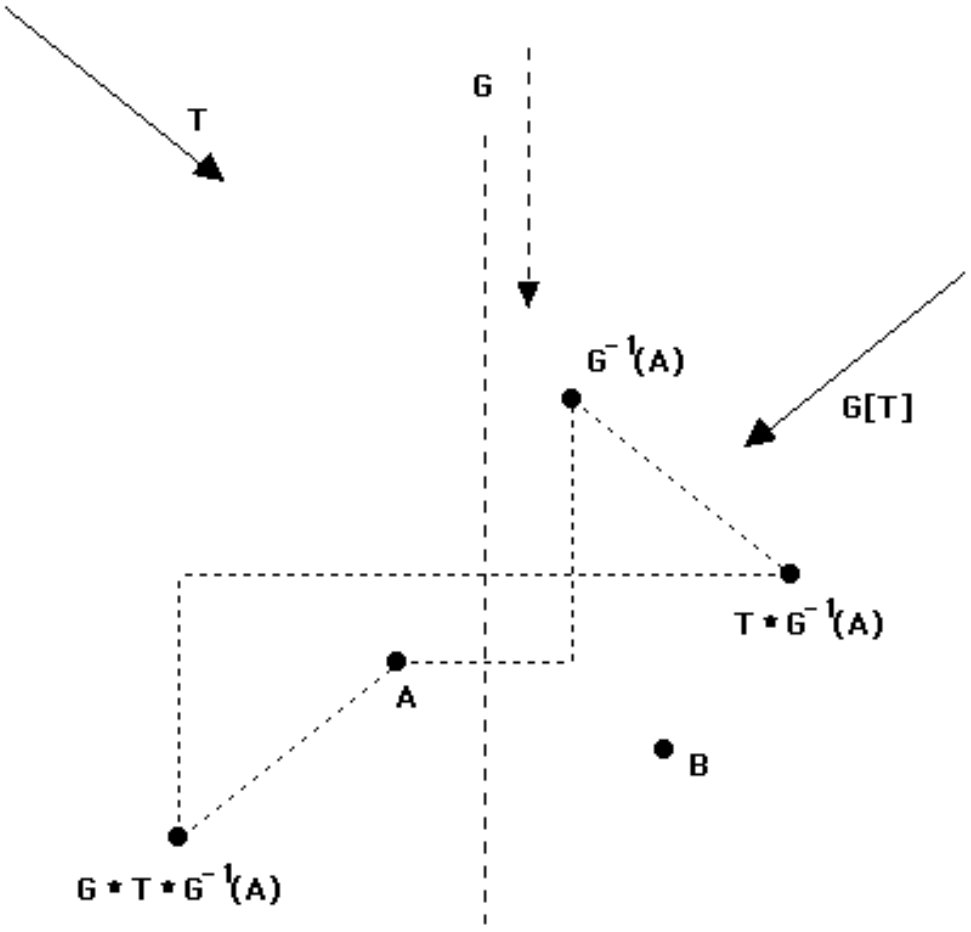
(1) The purpose of the diagram below is to illustrate the identity $\mathbf{T[G]} = \mathbf{T}*\mathbf{G}*\mathbf{T}^{-1}$: indeed you can see how the sequential application of translation $\mathbf{T}^{-1}$, glide reflection **G**, and translation **T** to the point A is equivalent to applying the glide reflection **T[G]**, the *image* of the glide reflection **G** under the translation **T**.

Repeat the process to the given point B.

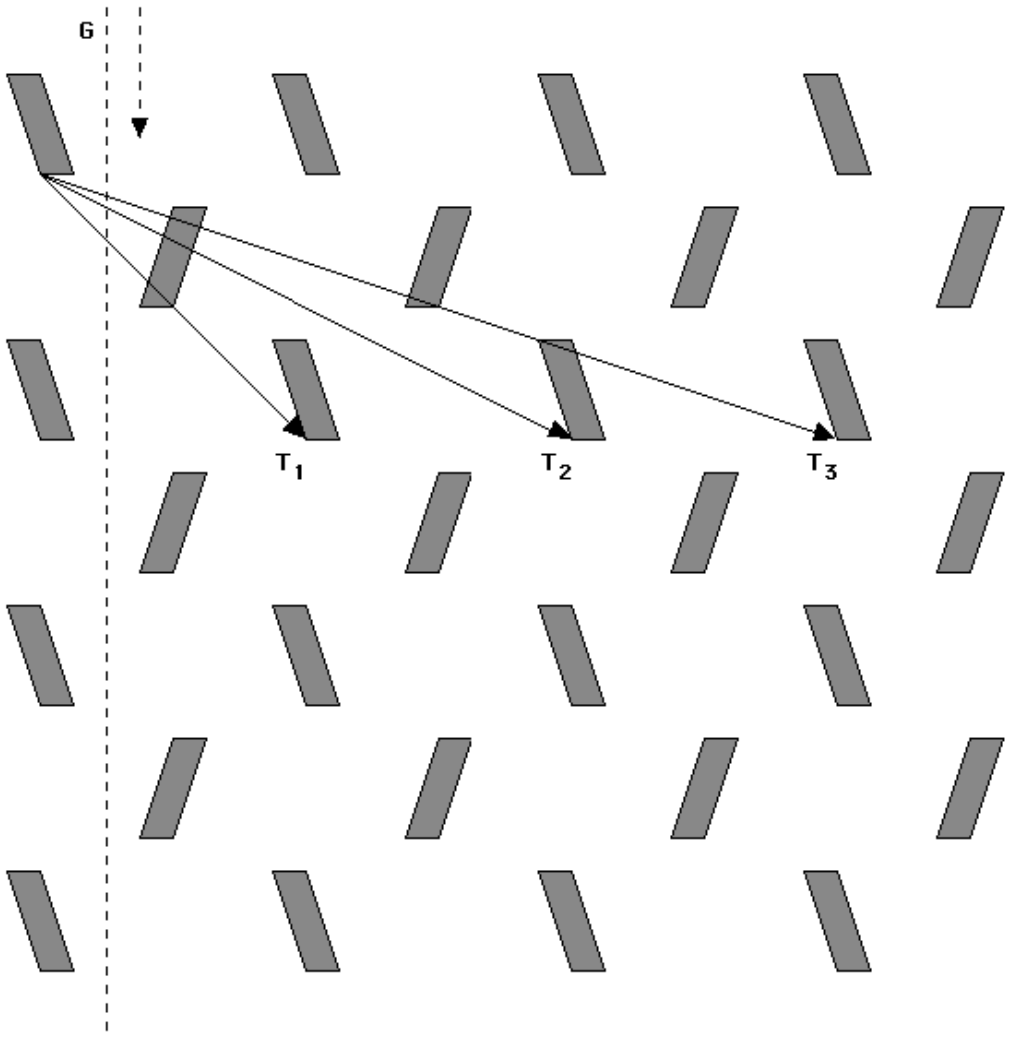


(2) The purpose of the diagram below is to illustrate the identity **G[T]** = **G**∗**T**∗**G**$^{-1}$: indeed you can see how the sequential application of glide reflection **G**$^{-1}$, translation **T**, and glide reflection **G** to the point A is equivalent to applying the translation **G[T]**, the *image* of the translation **T** under the glide reflection **G**.

Repeat the process to the given point B.

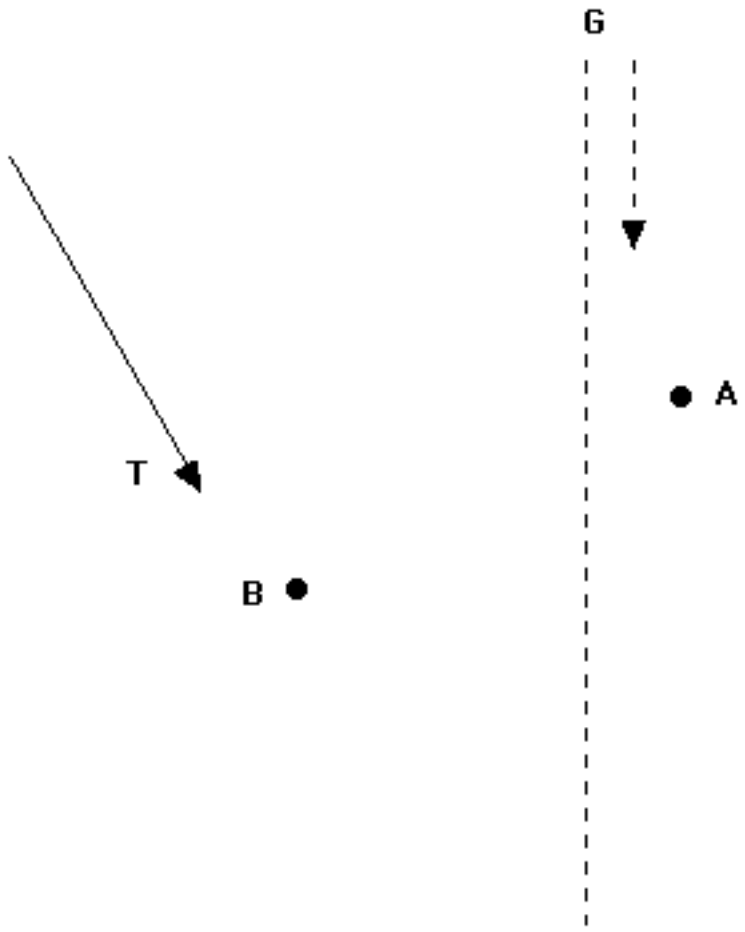


(3) Determine the following glide reflections: $\mathbf{T_1}*\mathbf{G}$, $\mathbf{T_2}*\mathbf{G}$, $\mathbf{T_3}*\mathbf{G}$, $\mathbf{T_1[G]}$, $\mathbf{T_2[G]}$, $\mathbf{T_3[G]}$.



What observation could you make based on your answers above?

(4) Determine the composition **G**∗**T** of the translation **T** followed by the glide reflection **G** empirically, by finding **G**∗**T**(A) and **G**∗**T**(B).



(5) Show, using the unit A, that the horizontal translation **t** may be obtained in (at least) two different ways as composition of other isometries: $\mathbf{G_2}*\mathbf{G_1^{-1}}$ ($\mathbf{G_1^{-1}}$ followed by $\mathbf{G_2}$) and $\mathbf{G_3^{-1}}*\mathbf{T}*\mathbf{G_1^{-1}}$ ($\mathbf{G_1^{-1}}$ followed by **T** followed by $\mathbf{G_3^{-1}}$). Are the two methods really different? Comment as appropriate.