Changing Technology Implies Changing Pedagogy

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Sam looked up from the stack of orders on his desk and glanced at his watch. 3:30, time to work on his project with Andrew. He pushed the orders to one side and turned to his computer. No picture this time, but Andrew's voice came through with sounds of students playing frisbee in the background.

Sam, 28, was a non-traditional student, fitting his course work around his work schedule. Andrew was a traditional first-year student. The two had been partners now for four weeks – though they had never met in person.

The background rock music ceased, and Sam heard Andrew's voice, "Hey Sam! What's it like in the real world today?"

"Usual thing, pushing paper… Better watch out for wild frisbees," Sam replied

"Right. But it's too nice to stay inside," Andrew explained.

"You kids have a soft life," Sam teased. "OK, let's get started."

The instructor's discussion of the project flashed up on the screen. It was just text – Professor Rodriguez was not much for adding voice descriptions. Not like Sam's political science prof, who always added a video stream with her verbal instructions.

Sam proposed a plan of work. "OK, we need to find a picture of a cross-section of a chambered nautilus, then construct a model of shell's spiral curve. And then we compare it with the real thing. Why don't you search the Web for a good picture, while I look through our class notes for the right formulas?"

Andrew agreed, and his end of the connection went dead. Sam entered a search query, refined it, and found what he wanted. He opened a computer algebra worksheet, made some notes, copied in code, modified it, and produced a test graph.

Andrew's voice returned, and a great picture of a shell appeared in the communication window. "I've put in the x- and y-axes. You can see. And here is a table of coordinates that I pulled off the picture with that cursor widget. … Oh great, you are all set with the modeling function. Right, exponential growth. I worked through that lesson last week. Now how do we match that up with the coordinate data?"

After another fifteen minutes of trial and error and a return to the class notes, the graph of the model function fit well – except for a stretch near the center that just wasn't the same as the rest of the spiral growth pattern.

"Let's ask Rodriguez about this center stuff," suggested Andrew. They quickly drafted a question, attached the picture and the worksheet, and e-mailed the lot to their instructor.

"Once we hear back from Rodriguez, we need to write up the report. Should be able to wrap this up in another hour," summarized Andrew.

Sam heard the rock music resume and then the dull thunk of a wayward frisbee catching Andrew in the head as he bent over his wireless notebook. The communication screen vanished just as Sam's boss showed up at his desk with another pile of orders.

Our scenario is only partly fanciful. For over five years we have had students working on a project similar to this – albeit in a classroom environment with help available from the instructor. The team project in the scenario could have come from our Equiangular Spiral module (Moore, *et al*., 2000) with some minor changes. (For example, we continue the project into its calculus implications, and we supply the picture.) Indeed, if Andrew did a Google image search for "chambered nautilus," he would have found over 200 great pictures, one of which is the one we used. And if he searched for "spiral" at the math.duke site (not a likely choice, to be sure), he would have found about 20 images, one of which is the picture on which we ask students to do their measurements.

Note in particular the following features of the scenario:

- Assigned group work
- Remote collaboration
- Use of the World Wide Web as an integral part of the project
- Traditional and non-traditional students working together in real time
- Time on task outside of classroom hours, but with (asynchronous) contact with the instructor

Students with notebook computers connected to a campus backbone by wireless cards are increasingly common. Extensive use of communication technology such as *NetMeeting* is less common but should be the norm in a couple of years. And, if this were an interactive, online article, we could provide a live link to video of students working through our module.

If our scenario is an accurate glimpse of the future – and we believe this future is almost upon us – what are the issues for student learning? We will discuss the following issues in this paper:

1. Learning and working in an increasingly rich technological environment
2. Making sense of mathematical information – using technology to check
3. Student-to-student interactions
4. Creation of interactive learning materials
5. Intellectual demands of these new forms of learning

1. Learning and working in an increasingly rich technological environment

Technology is changing they way students approach learning. Increasingly, they will conceive of their work in terms of interactive learning materials, computer algebra systems, spreadsheets, and web-based cooperation – with occasional use of pencil and paper. Learning how to learn in this environment is as important as learning about the mathematics itself.

Of course, technology has changed how we work and think about work in many ways. Let us illustrate with an example. Suppose you are thinking about writing a paper. You have a couple of ideas; possibly you jot them down on a pad. Then you want to expand them, so you make some more notes, circle them, and draw an arrow to the spot where they should be inserted. Reading that change to be inserted, you realize that other sentences need to be changed as well, and so on. Soon you have several sheets covered with words, lines, loops, and arrows that look more like an abstract painting than a draft of a paper. You quickly abandon paper and resort to a word processor to straighten things out. The point is not so much that you eventually used the technological tool, but that right from the beginning you were framing your thoughts about the paper with the use of the word processor in mind. Technology has changed the way you conceived of the task, as well as the way you carried it out.

Just as technology has changed the way that most of us approach a writing task, it also is changing the way students think about mathematical activities and carry out mathematical investigations. Graphs are now easy to display and can guide an investigation rather than just be an end product of a difficult calculation. With a symbolic calculating system, long trial calculations are also relatively easy and can also serve to guide an investigation. Similarly, data can be gathered, plotted, and compared. Now the important issues become what calculations, graphs, and data to display and how to interpret them.

2. Making sense of mathematical information – using technology to check

While it is true that technology will enable students to work with their favorite mathematical representations – symbolic, graphical, numeric – it is even more true that students will need to learn how to work and think productively, using many different modes of representation. Indeed, learning how to work and think in multiple representational modes may be one of the most important learning goals of mathematics courses in the age of technology.

In the old pencil-and-paper days, each calculation was likely to be long and subject to errors. Checking, if it was done at all, was likely to consist of performing the same calculation over again – probably making the same error. Now, complicated calculations are easy, and, more importantly, many new ways of checking are readily available. One can compare the symbolic derivative with a difference quotient calculation, a symbolic integration with a numeric integration, or a model function with data. Indeed, modeling provides a strong incentive for students to check their work and correct their mistakes. A student who is not bothered by a pencil-and-paper calculation of a negative volume is much more unsettled by a graph of a model function that does not lie anywhere near the data.

Since students have less emotional commitment to a short computer algebra system calculation than to a long pencil-and-paper one, they are more willing to check the result. They are not looking at the possibility that another 15-minute calculation will have to be repeated. With the pain of checking largely mitigated, the teacher is free to make checking a requirement – and to build checking strategies into the content of the course. Think of the consequences: Getting a confirmed right answer every time will be a normal expectation for both teachers and students. That means we will have to abandon the bell-shaped grading curve – which was never a scientifically sound idea anyway. But it also means – if we have the will – we can eliminate high withdrawal/failure rates and turn mathematics into a subject in which students expect to succeed.

The National Research Council study *How People Learn* (Bransford, *et al*., 1999; Donovan *et al*., 1999) identifies self-monitoring as one of the key findings from research about successful learning. Specifically (Donovan *et al*., 1999, p. 13), "A 'metacognitive' approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them." The concept of confirming every mathematical calculation is a local implementation of this principle, since most students start most assigned tasks with the goal of getting the right answer.

This self-monitoring function, known to be important for learning in general, takes on added importance with ubiquitous Web access. Using the Web, a student may find many others who have already dealt with the problem under consideration. How can one know which calculations or conclusions to trust? The ability to evaluate information, to decide what is reasonable, what is correct, is vital to making intelligent use of web resources.

One illustration of this is Sugg (1996), a page of lecture notes for a mathematically oriented biology course. This page contains a lot of apparently correct and useful information, but it draws an incorrect mathematical conclusion – one that is obvious to a mathematician but that would easily fool a student. Specifically, Sugg analyzes the classical Lotka-Volterra predator-prey model (in its differential form, not a discrete model) and concludes that the model is inherently unstable. Nowhere on the page is there any hint of the population cycles that are the correct trajectories of the differential system – and also the observations from nature that motivated both Lotka and Volterra.

Another related issue, one that is general across the sciences and engineering, is created by simulations that eliminate the need to perform real physical experiments. This becomes increasingly important in mathematics as modeling becomes a central part of mathematics courses. The issue is not just the accuracy of the simulation, but also the student's conception of the physical world. What models and data are being used to create a given simulation? How reasonable is it that the simulation accurately represents the aspect of the world under consideration? How can one check?

3. Student-to-student interactions

A particularly important challenge of this new environment will be designing learning experiences that support cooperative work and the development of a class-wide community of learners. One way to go about this is described in (Smith, 2001) in the context of a differential equations course – but the same principles could be expected to work with lower-level courses as well.

As we imagined in our opening scenario, there will be great opportunities for productive cooperative work – even for students with little or no opportunity for face-to-face contact. In addition to Microsoft's *NetMeeting* (http://www.microsoft.com/windows/netmeeting), many other ways to accomplish real-time collaborations are now available. Some of the other examples include

- Blackboard (http://www.blackboard.com),

- WebCT (http://www.webct.com),
- Netopia's *Timbuktu* (http://www.netopia.com/en-us/software/products/tb2/),
- AT&T's *Virtual Network Computing* (http://www.uk.research.att.com/vnc/),
- Interwise's *Enterprise Communications Platform* (http://www.interwise.com/).

The capabilities of these products are all different from one another, as are their prices, but each enables collaborators to share work in real time via the Internet.

In the other direction, there is a tendency for technology to provide the individual with a personal learning environment, insulated from contact with others. With headphones delivering a stream of background music and individual hand-held computing devices replacing workstations that can accommodate two people, the individual student may retreat from any significant learning interaction. It will be important for both curriculum developers and instructors to focus on this issue.

4. Creation of interactive learning materials

What are the implications of technology for developers of learning materials? In the recent past, individual faculty have been creating interactive class materials shortly before they were needed in class. Then, more often than not, the materials were left alone until the next time the author-instructor was teaching the same course. Even if an author did more work, it was unlikely that the materials were ever "finished" in any reasonable sense. In some ways, this is comparable to the period in the 70's when many individuals wrote their own word processing programs. After a short transition period, users came to expect more from a word processing program than most individuals were willing or able to produce. Now most of us use one of the common commercial programs.

For learning materials, there are currently two trends. One is for teams of individuals to work together to produce materials that include sophisticated interactions delivered in a setting that is easy to use and very flexible. The other trend is similar to the phenomenon of open-source software. Authors cooperate in a loose federation that combines compatible learning components in different ways as necessary and leaves the product for further development by others.

In the old textbook-oriented model, a small group of authors, working very intensely, produced most of the major text material. The individual faculty member's responsibility was to create a syllabus around the published text. Now, regardless of the interactive materials used, the instructor is going to be much more closely involved, often adapting the materials for his or her own use. Beyond that, many more instructors will be part of the design and development of the materials. However, if it is done well, the development of learning materials that incorporate technology will take extensive time and effort. How are authors to be rewarded? The rewards will probably not be royalty income so much as scholarly recognition. So far, this sort of recognition has been slow to develop.

5. Intellectual demands of these new forms of learning

Finally, we need to be clear that students will be expected to do more challenging tasks than in the past – particularly in precalculus and calculus courses. In the past, just deciding on a symbolic calculation algorithm and executing it with care represented a satisfactory learned response. Now the student will need to recall and evaluate the usefulness of and connections among a variety of representations and computations. This is a higher-order intellectual activity – one that will allow learning at a deeper level. Fortunately, reforms such as the NCTM *Principles and Standards* (NCTM, 2000) have paved the way for this change.

It is no longer acceptable to assess student learning by asking them to solve calculational problems because computer algebra system (CAS) capabilities are widely available to almost everyone. For example, the Texas Instruments TI-89 (about $150) provides powerful algebra and calculus capabilities (with 2-D and 3-D graphics) in a hand-held calculator. Many schools and colleges provide site-license access to *Maple*® or *Mathematica*®. *StudyWorks*® (essentially a fully functional version of *Mathcad*®) is available from Mathsoft for about $40. And there are a number of free or inexpensive online services that will accept a problem input and provide the output from, say, *Maple*® or *Mathematica*®. One example is *The MathServ Calculus Toolkit* at Vanderbilt University (Crooke and Tschantz, 2002), which includes a number of precalculus topics as well. Simply forbidding the use of any of these tools is about as effective as sticking a finger in a crumbling dike.

In fact, it never did make sense to assess student understanding of mathematics solely or primarily by their ability to do unaided symbolic calculations. At best this ability is a poor proxy for understanding, as anyone can learn simply by asking students to explain what they are doing as they carry out a calculation. And generations of students have come to believe that the calculations are what mathematics *is*. Worse, reserving the rewards for those who are proficient at calculations in a timed, closed-book, no-technology test setting has denied success to many other students who are quite capable of understanding mathematical concepts – as we have learned by teaching those students in technology-rich environments. Whatever the limitations on our profession in the past, we are not condemned to repeat failing practices forever.

On the positive side, a recent analysis and synthesis (Heid and Blume, in press) of research on the use of technology in mathematics instruction at all levels has documented strong support for welcoming technology as a component of our pedagogical practices. One of us co-authored the calculus chapter (Tall, *et al*., in press) in this volume, which includes among its conclusions the following:

- "Technology integrated intelligently with curriculum and pedagogy produces measurable learning gains. …
- "There is evidence that using tools such as *Mathematica* and *Maple* for conceptual exploration … leads to conceptual gains in solving problems that can transfer to later courses. In comparison, students following traditional courses tend to use more procedural solution processes.
- "Technology enables some types of learning activities (e.g., discovery learning) and facilitates some others (e.g., cooperative learning) that are harder or impossible to achieve without technology."

Of course, the completed research all refers to technologies that have been available in the past. The technologies becoming available to us now hold promise for even more exciting gains – if we can keep up with the intellectual challenge of adapting our pedagogies to the realities of the world in which our students live.

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