How The Error Changes?

Power Laws For Estimation

The error expected in estimating the mean will decrease with the number n of random points, in the following ways:

\[ \text{Error} \propto n^{-0.5} \]

Example: Find Type 1 and Type 2 estimates, using pseudo-random numbers for area under the curve of \( y = x^2 \) in \([0,1]\).

For type 1, we can get \( n \) uniform random numbers \( x \) in \([0,1]\) and find the average, the differences of average value and the correct answer 1/3.

For type 2, we can get \( n \) uniform random numbers \( x \) in \([0,1]\) and check whether or not under the curve and find the differences of correct answer and statistic answer.

Random Numbers and their Applications

The Concept of Random Number

- Strive to be random in the sense of being independent and identically distributed.
- A Random Number Generator (RNG) is a computational device designed to generate a sequence of numbers.
- The string of values generated by such algorithms is generally determined by a fixed number called a seed.

Linear Congruential Generator

One of the most common RNG is the linear congruential generator:

\[ X_i = aX_{i-1} + b \mod m \]
\[ U_i = X_i / m \]

Example: Approximate the area under the curve \( y = \sin(15x) \) in \([0,\pi]\).

The area is the mean value of function times \( \pi \).
By using the random number in \([0,\pi]\), we can calculate each \( f(U_i) \) and the function average.

Example: Find the area that satisfy

\[ (2X-1)^8 + 8(2y-1)^8 < 1 + 2(2y-1)^3(3X-2)^3 \]

Given \((x,y)\), we can easily check whether or not in the set. By using the random number in \([0,1]\), we can get a pair of number \((U_i, U_{i+1})\) and check if they satisfy.

Random Walk

A random walk \( W(t) \) is defined on the real line by starting at \( W(0) \) and moving a step of length \( S(i) \) at each integer time \( i \), where the \( S(i) \) are independent and identically distributed random variable.

\[ W(t) = W(0) + S(1) + S(2) + \ldots + S(t) \]

Example: Use a Monte Carlo Simulation to approximate the probability that the random walk exits the interval \([-3,6]\) through the top boundary 6. Carry out \( n=10000 \) random walks.